Overlapping PPM Fiber-Optic CDMA Systems with Imperfect Code Synchronization

Anh T. PHAM[†a], *Student Member and* Hiroyuki YASHIMA[††], *Member*

SUMMARY   This paper theoretically evaluates the performance of overlapping pulse-position modulation (OPPM) fiber-optic code-division multiple-access (FO-CDMA) systems in the presence of code synchronization errors. The analysis is carried out with a constraint on throughput-pulsewidth product. Discussions on effects of various system parameters, such as timing offset, index of overlap, number of users, are presented. The results show that the OPPM FO-CDMA systems with high index of overlaps have better resistance against imperfect synchronization. In fact, the acceptable performance could be maintained even with timing offsets of up to 30% of chip pulsewidth. On the other hand, strict code synchronization is necessarily required, preferably within a half code chip pulsewidth.
key words: *optical code-division multiple-access, overlapping pulse-position modulation, optical orthogonal code, code synchronization*

1. Introduction

Recently, in the domain of local area and access networks, FO-CDMA has been considered as a promising multiple-access method [1]. FO-CDMA systems with on-off keying (OOK), pulse-position modulation (PPM) and overlapping PPM (OPPM) have been introduced recently, of which OPPM FO-CDMA scheme is the most favorable [2]–[4]. It is due to two main reasons. First, OPPM retains the advantage of PPM in terms of implementation simplicity (no thresholder is required). Secondly, unlike PPM, OPPM could improve the throughput (bits/s) while keeping pulsewidth unchanged.

   The block diagram of an OPPM FO-CDMA transmitter with signal format is described in Fig. 1. From the information source, M-ary data symbols are sent out. These data symbols are used to modulate the position of laser pulse to form the OPPM signal. The OPPM FO-CDMA signal is generated by imprinting a spreading sequence, with good correlation properties, onto the OPPM signal. An example of OPPM FO-CDMA signal format is also depicted in Fig. 1 with multiplicity $M = 7$, and index of overlap $\gamma = 3$; the spreading sequence has length $L = 9$ and weight $w = 2$; T denotes the frame width. The allowable spreading interval of each OPPM symbol, which is usually called slot, is τ, and T_c denotes the width of an OPPM FO-CDMA chip pulse. At the receiving end, before transmitted data is de-
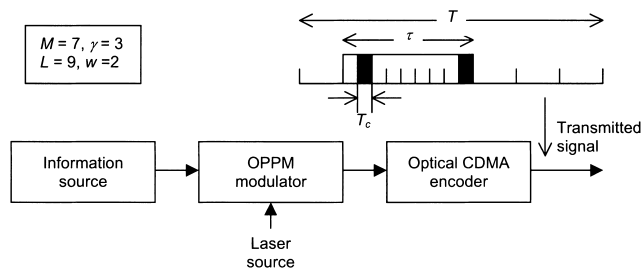
Fig. 1   Block diagram of an OPPM FO-CDMA transmitter.

tected and recovered (by the photodetector and OPPM decoder), the desired optical pulses are collected by correlating the received signal (including signal from all users) with a local replica of the spreading sequence, which characterizes the desired user. This correlation requires a strict synchronization between the receiver and the desired signal's spreading sequence so that pulse collection could be exact and complete. The perfect synchronization is however not always available. The effect of imperfect code synchronization on OOK and PPM FO-CDMA systems was presented by [5] and [6]; however, it is, to our knowledge, not clarified for OPPM systems.

   The aim of this paper is, thus, to evaluate the performance of OPPM FO-CDMA systems in the presence of code synchronization errors. For the evaluation, shot-noise-limited photon-counting receiver is assumed while dark currents are neglected because their effect is minor [2]. In the next section, we will theoretically derive the upper bound on bit error probability in the presence of synchronization errors. Numerical results and discussions are presented in Sect. 3 before we give our conclusions in Sect. 4.

2. Theoretical Analysis

For the sake of mathematical simplicity, we assume all users are frame synchronous. It is important to note that this assumption is not affected by code synchronization errors between local replica and the spreading sequence of the desired user. Furthermore, we assume optimum optical orthogonal codes (OOC), i.e. their off-peak auto-correlation and cross-correlation are bounded by only one [7], as spreading sequence for the evaluated system. Due to these assumptions, each user can interfere with the desired user at either only one or zero pulse position. We define the interference state vector as

$$\kappa_0^M \equiv \{\kappa_0, \kappa_1, \dots, \kappa_{M-1}\}, \tag{1}$$

where κ_i, $i \in \{0, 1, \dots, M-1\}$, is a random variable that represents the number of users causing interference to the desired user within slot i-th of its transmitted frame. The probability that there are l_i ($0 \le l_i \le N-1$) users that cause interference to i-th slot of the desired user can be expressed as [2]

$$P_r\{\kappa_i = l_i\} = \binom{N-1}{l_i} p^{l_i}(1-p)^{N-1-l_i}, \tag{2}$$

here N is number of users, and $p = \frac{\gamma w^2}{ML}$, where L and w are length and weight of OOC.

The decision on transmitted data is made by an OPPM demodulator based on a comparison of photon counts over slots of the OPPM frame. Error occurs when the photon counts over the targeted slot is not the highest one. We denote Y_i, $i \in \{0, 1, \dots, M-1\}$, as the photon counts over slot i-th of OPPM frame. Under the assumption that the transmitted data is equally likely, the probability of error at frame level can be written as

$$
\begin{aligned}
P_e &= P_r\{Y_j \ge Y_i, \text{ some } j \ne i | D = i\}.P_r\{D = i\} \\
&= P_r\{Y_j \ge Y_0, \text{ some } j \ne 0 | D = 0\}, \tag{3}
\end{aligned}
$$

here D is a random variable that denotes the transmitted data symbol. The upper bound of this probability of error can be given by

$$
\begin{aligned}
P_e &= P_r\{Y_j \ge Y_0, \text{ some } j \ne 0 | D = 0\} \\
&= \sum_{l_0^{M-1}} P_r\{Y_j \ge Y_0, \text{ some } j \ne 0 | D = 0, \\
&\quad \kappa_0^{M-1} = l_0^{M-1}\} P_r\{\kappa_0^{M-1} = l_0^{M-1}\} \\
&\le \sum_{l_1^{M-1}} P_r\{Y_j \ge Y_0, \text{ some } j \ne 0 | D = 0, \kappa_0 = 0 \\
&\quad \kappa_1^{M-1} = l_1^{M-1}\} P_r\{\kappa_1^{M-1} = l_1^{M-1}\}. \tag{4}
\end{aligned}
$$

Calculation of this probability of error involves comparing the photon counts over slot 0 and the photon counts over the remaining slots. We note that there are at most $\gamma - 1$ in remaining $M - 1$ slots where self-interference (i.e. interference caused by slot 0) may occur. Denote ν_j as number of pulses (from slot 0 of the desired user) that causes self-interference in the j-th slot, ν_j could be either 0 or 1 as optimum OOC was assumed. Employing union-bound for (4) we have

$$
\begin{aligned}
&P_r\{Y_j \ge Y_0, \text{ some } j \ne 0 | D = 0, \kappa_0 = 0, \kappa_1^{M-1} = l_1^{M-1}\} \\
&\le (M - \gamma) P_r\{Y_1 \ge Y_0 | D = 0, \kappa_0 = 0, \kappa_1 = l_1, \nu_1 = 0\} \\
&\quad + \sum_{j=1}^{\gamma-1} P_r\{Y_j \ge Y_0 | D = 0, \kappa_0 = 0, \kappa_j = l_j, \nu_j\} \\
&\le (M - \gamma) P_r\{Y_1 \ge Y_0 | D = 0, \kappa_0 = 0, \kappa_1 = l_1, \nu_1 = 0\} \\
&\quad + (\gamma - 1)[q P_r\{Y_1 \ge Y_0 | D = 0, \kappa_0 = 0, \kappa_1 = l_1, \nu_1 = 1\} \\
&\quad + (1 - q) P_r\{Y_1 \ge Y_0 | D = 0, \kappa_0 = 0, \kappa_1 = l_1, \nu_1 = 0\}], \tag{5}
\end{aligned}
$$

where $q \equiv P_r\{\nu_1 = 1\}$, which, under the assumption of uniform distribution of marks in the spreading sequence, can be given as

$$q \equiv P_r\{\nu_1 = 1\} = \frac{w(w-1)}{L-1}. \tag{6}$$

From (4) and (5), the error probability at the frame level P_e can be derived as

$$
\begin{aligned}
P_e \le \sum_l \Big[&(M - 1)P_r\{Y_1 \ge Y_0 | D = 0, \kappa_0 = 0, \kappa_1 = l, \\
&\nu_1 = 0\} + (\gamma - 1)q[P_r\{Y_1 \ge Y_0 | D = 0, \kappa_0 = 0, \\
&\kappa_1 = l, \nu_1 = 1\} - P_r\{Y_1 \ge Y_0 | D = 0, \kappa_0 = 0, \\
&\kappa_1 = l, \nu_1 = 0\}] \Big], \tag{7}
\end{aligned}
$$

Assuming a shot-noise-limited photon-counting receiver for the evaluated system, photon counts over slots are thus only originated from desired user and multiple-access interference (MAI). Denoting Z_i and I_i respectively as photon counts originated from the desired user and MAI to the i-th slot, we have

$$Y_i = Z_i + I_i. \tag{8}$$

Here we further note that, in principle, before the receiver actually starts operation, the coarse alignment status, where there is only small fraction (usually within a chip pulse) between local replica and desired signal's spreading sequence, must be achieved by the code acquisition system. We, therefore, only consider code synchronization errors of within a chip pulsewidth, i.e. $|\Delta T| \le T_c$. It means, timing offset $\epsilon = \frac{|\Delta T|}{T} \le 1$. As depicted in Fig. 2, both Z_i and I_i are affected by code synchronization errors. The average value of photon counts originated from the desired user, $E[Z_i]$, can be easily obtained as

$$
E[Z_i | D] = \begin{cases} (1 - \epsilon)T_c \lambda_s w, & \text{if } D = i \\ 0, & \text{otherwise} \end{cases} \tag{9}
$$

where λ_s is photon absorption rate, which is assumed to be the same for all users, and T_c is chip pulsewidth.

Without loss of generality, we consider only $\Delta T \ge 0$, i.e. synchronization errors cause local replica to shift right, as depicted in Fig. 2. We realize that I_i contains both interfering pulses to the desired user's marked position and the adjacent position. The average value of I_i thus can be expressed as
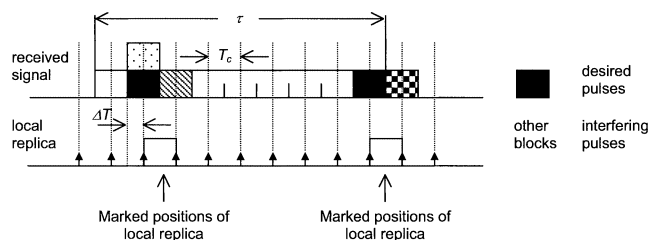

Fig. 2 Effect of imperfect code synchronization.

$$E[I_i|D] = (1 - \epsilon)T_c\lambda_s(\kappa_i + \nu_j) + \epsilon T_c\lambda_s(\kappa_i^{(-1)} + \nu_j), \quad (10)$$

here $\kappa_i^{(-1)}$ denotes a random variable that represents the number of interfering users at the right-shifted positions of the desired user's marks in the i-th slot; ν_j is a random variable that represents self-interference as mentioned previously. Assuming $\kappa_i = \kappa_i^{(-1)}$ could result in a looser bound in calculation of the error probability; $E[I_i|D]$ then can be written as

$$\begin{aligned} E[I_i|D] &= (1 - \epsilon)T_c\lambda_s(\kappa_i + \nu_j) + \epsilon T_c\lambda_s(\kappa_i^{(-1)} + \nu_j) \\ &= (1 - \epsilon)T_c\lambda_s(\kappa_i + \nu_j) + \epsilon T_c\lambda_s(\kappa_i + \nu_j) \\ &= T_c\lambda_s(\kappa_i + \nu_j). \end{aligned} \quad (11)$$

From (7), (8), (9), (11) and noting that shot-noise-limited photon-counting receiver was assumed, we can derive the error probability at frame level as a function of timing offset ϵ as

$$\begin{aligned} P_e(\epsilon) \le\ & (N - 1) \sum_{l=0}^{N-1} \binom{N-1}{l} p^l(1-p)^{N-1-l} \\ &\times \sum_{k_1=0}^{\infty} Pos(k_1, K_{10}) \sum_{k_0=0}^{k_1} Pos(k_0, K_0) \\ &+ (\gamma - 1)q \sum_{l=0}^{N-1} \binom{N-1}{1} p^l(1-p)^{N-1-l} \\ &\times \sum_{k_1=0}^{\infty} [Pos(k_1, K_{11}) - Pos(k_1, K_{10})] \\ &\times \sum_{k_0=0}^{k_1} Pos(k_0, K_0) \end{aligned} \quad (12)$$

here, $K_{11} = T_c\lambda_s(l+1)$, $K_{10} = T_c\lambda_s l$, $K_0 = (1-\epsilon)T_c\lambda_s w$, p and q were defined previously, and

$$Pos(k, K) \equiv e^{-K}\frac{K^k}{k!}$$

Finally, bit error probability, with timing offset ϵ, can be obtained from

$$P_b(\epsilon) = M/2(M-1)P_e(\epsilon). \quad (13)$$

3. Numerical Results

For an insightful evaluation of the effect of timing offset on performance of the evaluated system with variation of various parameters, such as index of overlap, number of users, code length and weight, we hold both data rate (throughput) of each user R_T and pulsewidth T_c fixed. A throughput-pulsewidth product, denoted as R_0, is defined to represent these two parameters as $R_0 \equiv R_T T_c$. For the overlapping PPM FO-CDMA system, throughput R_T can be expressed as

$$R_T \equiv \frac{\log M}{T} = \frac{\gamma \log M}{M\tau} = \frac{\gamma \log M}{MLT_c} \quad \text{(nats/s)}. \quad (14)$$
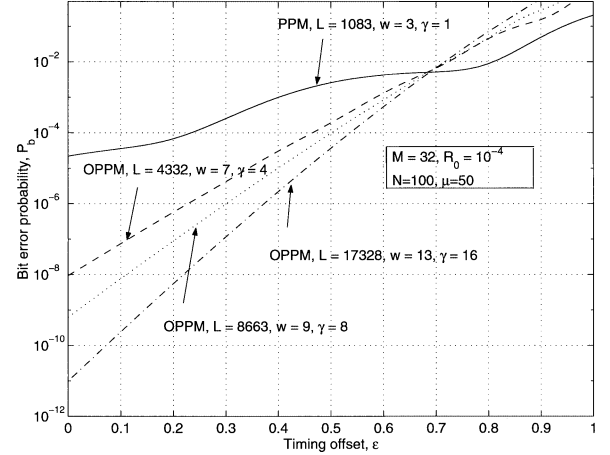

Fig. 3 Bit error probability versus timing offset ϵ.

The throughput-pulsewidth product thus can be written as

$$R_0 = \frac{\gamma \log M}{ML} \quad \text{(nats/chip)}. \quad (15)$$

As we are interested in the effect of γ, we first select several values of γ with a fixed M. From (15) L can be chosen. For a fixed number of users N, code weight w then can be decided in accordance with the OOC constraint as [7]

$$N \le \frac{L-1}{w(w-1)}. \quad (16)$$

Figure 3 shows the bit error probability versus timing offset ϵ. Number of users N is 100, M is set to 32, average photon counts per nat $\mu = 50$, and R_0 is fixed at 10^{-4} for all users. Different values of γ are selected (noting that when $\gamma = 1$ we have a PPM system) and L is chosen accordingly to (15). Finally, maximum w is selected to satisfy (16). Average photon counts per pulse can be computed as $T_c\lambda_s = \mu \log M/w$. It is seen that when ϵ increases (from 0 to 1), the system performance is severely degraded. Actually, when ϵ is higher than 50% of the chip pulsewidth, in all cases P_b exceeds 10^{-6}, which is considered the limit for acceptable performance. On the other hand, we can see that with high index of overlaps, the system performance is still acceptable with timing offsets of as high as 30% despite the fact that P_b increases faster with higher index of overlaps. For example, for an OPPM with $\gamma = 16$, $L = 17328$ and $w = 16$, we can achieve bit error probability of $P_b = 1.1537 \times 10^{-7}$ with $\epsilon = 0.3$.

Next, in Fig. 4, we evaluate the system performance versus average photon counts per nat for timing offsets $\epsilon = 0.1$ and 0.3. The throughput-pulsewidth R_0 is also fixed at 10^{-4}, other parameters are the same as in Fig. 3. For $\epsilon = 0.1$, the OPPM system can basically achieve the acceptable performance ($P_b < 10^{-6}$) when average photon counts per nat is higher than 20. For instance, with $\gamma = 8$, $L = 8663$, $w = 9$, $\mu = 20$, P_b can be as good as 2.562×10^{-7} when $\epsilon = 0.1$. Moreover, when μ increases, the performance
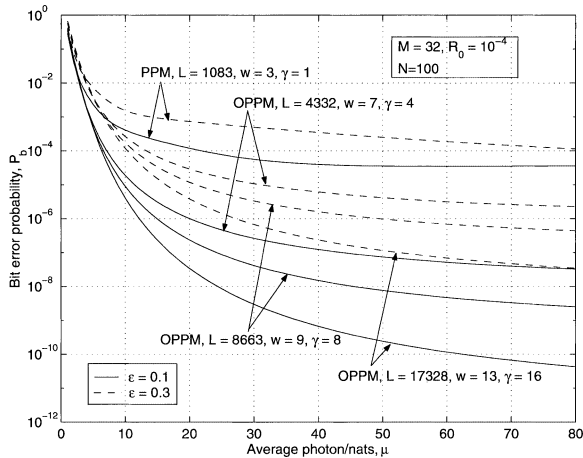
Fig. 4 Bit error probability versus average photon counts per nat, $\epsilon = 0.1$ and 0.3.
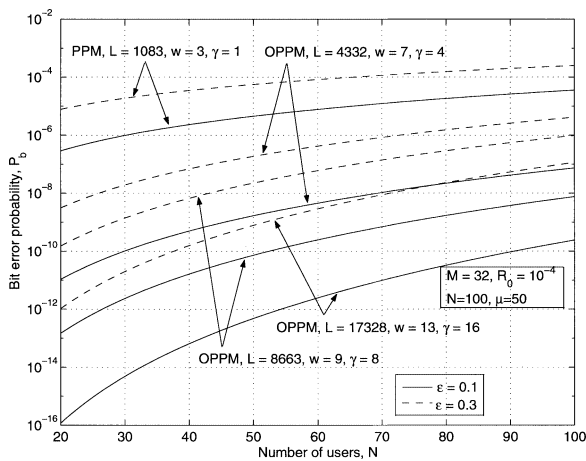

Fig. 5 Bit error probability versus number of users, $\epsilon = 0.1$ and 0.3.

of the system using OPPM could be notably improved, especially with higher index of overlaps. For example, when $\mu \geq 50$, most of the systems using OPPM could maintain a P_b of less than 10^{-6} with timing offsets of up to 0.3.

Finally, we evaluate P_b versus number of users N for timing offsets $\epsilon = 0.1$ and 0.3 in Fig. 5. We especially consider P_b with N less than 100, the maximum achievable number of users (as we selected w in accordance to inequality (16)). It is found that, we can always achieve the accept-

able performance for the evaluated system using OPPM with $N \leq 60$. While under the same constraint of throughput-pulsewidth product, acceptable performance could almost never be achieved by the system using PPM method ($\gamma = 1$).

4. Conclusions

We have theoretically evaluated the effect of code synchronization errors on the performance of OPPM FO-CDMA systems using OOC. The shot-noise-limited photon-counting receiver is assumed. The upper bound on bit error probability is derived in the presence of code synchronization errors, represented by timing offset parameter. We discuss bit error probability with a constraint on throughput-pulsewidth product. We find that strict synchronization is necessarily required; basically timing offset should not be higher than a half code chip pulsewidth. Under the constraint of throughput-pulsewidth, the OPPM systems with higher index of overlaps offer better resistance against synchronization errors. Actually, acceptable performance ($P_b < 10^{-6}$) could be achieved even with timing offsets of up to 0.3 when average photon per nat is high enough (e.g. 50 in the evaluated system).

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