Blending operations for the functionally based constructive geometry

Alexander A. Pasko Vladimir V. Savchenko

Abstract

A description of blending set-theoretic operations is proposed for geometric objects with implicit surfaces. We use modified R-functions as a continuous analytical description of a resultant solid. Intuitive shape control and aesthetic blend definition by hand drawn strokes are supported. An offsetting operation is applied in the particular case of constant-radius blending.

Introduction

The problem of joining several surfaces in a complex object with a smooth surface is called blending. Papers [8,9] contain detailed discussion of problems being encountered in blending. Let us mention here only the main difficulties and requirements summarized from papers [1-4, 6-9, 12]. Note that some items are overlapping or even contradictory to each other:

• Tangency of a blend surface with the base surfaces.
• Easy intuitive control of the blending surface shape.
• Necessity to perform for blended objects all the computations possiblemfor unblended objects including set-theoretic operations.
• Blend interference or ability to blend on blends and as the particular case complex vertices (or corners) blending.
• C^1 continuous blending function over the entire domain of definition.
• Blending definition of the basic set-theoretic operations: intersection, union and subtraction.
• Single edge blending or localizing a blend to a given region about intersection curve of two faces.
• Added and subtracted material blends.
• The ability to produce constant-radius blending.
• No restriction of circular cross sections, or in other words, the requirement of variable-radius blends.
• Exact representation for blends instead of an approximation.
• Automatic clipping of unwanted parts of the blending surface.
• Blending of two non-intersecting surfaces.
• Functional constraints.
• Aesthetic blends constrained by appearance.

Several methods have been proposed for blending objects defined by set-theoretic operations on primitives with implicit surfaces. The blending method of [6] is based on the approximations of min/max functions used for Boolean operations. Constant-radius blends [9] are defined using the concept of a rolling sphere and the offset operation over

solids. Blending of several algebraic surfaces has been described in [12] with the notion of geometric continuity applied to implicitly defined surfaces. Methods of potential blending [1,4] are based on the distance properties of base functions. The displacement method [8] provides the blending function differentiable at any point in the domain.

The need still exists in the continuous analytical description of the set-theoretic operations with controllable blend shape. Beginning with geometric solids represented exactly by continuous functions of three variables, we have tried to satisfy most of the above-mentioned requirements . We use the theory of R-functions to implement functionally based Constructive Solid Geometry (CSG). In this paper, we propose the modification of R-functions to obtain blending set-theoretic operations. Intuitive shape control and aesthetic blending defined by hand-drawn strokes are supported. Special attention is paid to the particular case of constant-radius blending.

The remainder of this paper is organized as follows. Section 2 gives the brief introduction to R-functions. Section 3 deals with blending set-theoretic operations. Section 4 is devoted to the procedure of generating an aesthetic blend defined by hand drawn strokes. Section 5 describes constant-radius blending.

Function representation and set-theoretic operations

We consider closed subsets of n-dimensional Euclidean space E^n with the definition:

$$f(x_1, x_2, ..., x_n) \geq 0 \qquad (1)$$

where f is a real continuous function defined on E^n. We will call f a *descriptive function*. The defining inequality (1) we call the *function representation* of a geometric object [5]. In the three-dimensional case the boundary of such an object is a so-called "implicit surface". The function can be defined in various ways: analytically, with an evaluation algorithm, or with tabulated values and an appropriate interpolation procedure. The major requirement of the function is to have at least C^0 continuity. Expression (1) allows the definition of closed n-dimensional objects in E^n space with the following characteristics:

$f(X) > 0$ - for points inside the object;

$f(X) = 0$ - for points on the object boundary;

$f(X) < 0$ - for points outside the object.

Here $X=(x_1, x_2, ..., x_n)$ is a point in E^n. Note that we do not use the term "implicit function" here. The definition of the object is the inequality (1) with the explicit function $f = f(x_1, x_2, ..., x_n)$ but not the implicit function $f(x_1, x_2, ..., x_n)=0$ that defines the boundary of the object.

Let binary operations on geometric objects be defined by:

$$F(f_1(X), f_2(X)) \geq 0,$$

where F - a continuous real function of two variables. Such operations are closed on the set of function representations. The output of one such operation can be used as an input to a subsequent one. We consider binary set-theoretic operations for which the resulting

object includes its boundary points. The analytical definitions of such operations have been studied by Rvachev [10,11] for solving problems of mathematical physics on areas with complex shape. It was proposed to transfer from the set-theoretic operations over areas with the description (1) to the operations over descriptive functions. The resultant object will have the descriptive function as follows:

$f_3 = f_1 \mid f_2$ for union;
$f_3 = f_1 \,\&\, f_2$ for intersection;
$f_3 = f_1 \setminus f_2$ for subtraction,

where f_1 and f_2 are descriptive functions of initial objects and $\{\mid, \&, \setminus\}$ are signs of so-called R-functions. One of the possible analytical descriptions of the R-functions is as follows:

$$f_1 \mid f_2 = \frac{1}{1+a}\,(f_1 + f_2 + \sqrt{(f_1^2 + f_2^2 - 2af_1f_2)}) \qquad (2)$$

$$f_1 \,\&\, f_2 = \frac{1}{1+a}\,(f_1 + f_2 - \sqrt{(f_1^2 + f_2^2 - 2a\, f_1f_2)})$$

where $a = a(f_1, f_2)$ - an arbitrary continuous function satisfying following conditions:
$-1 < a(f_1, f_2) \le 1$, $a(f_1, f_2) = a(f_2, f_1) = a(-f_1, f_2) = a(f_1, -f_2)$.
The expression for the subtraction operation is the same as that for intersection, except one replaces f_2 with $(-f_2)$. If $a = 1$ the functions (2) become

$$f_1 \mid f_2 = \max(f_1, f_2) \qquad (3)$$
$$f_1 \,\&\, f_2 = \min(f_1, f_2)$$

This is the particular case described in [6]. The functions (3) are very convenient for doing calculations but have C^1 discontinuity where $f_1 = f_2$. If $a=0$ the functions (2) take the form most useful in practice:

$$f_1 \mid f_2 = f_1 + f_2 + \sqrt{(f_1^2 + f_2^2)} \qquad (4)$$

$$f_1 \,\&\, f_2 = f_1 + f_2 - \sqrt{(f_1^2 + f_2^2)}$$

The functions (4) have C^1 discontinuity only in points where both arguments are equal to zero. If C^m continuity is to be provided, one may use another set of R-functions:

$$f_1 \mid f_2 = (f_1 + f_2 + \sqrt{(f_1^2 + f_2^2)})\,(f_1^2 + f_2^2)^{m/2} \qquad (5)$$

$$f_1 \,\&\, f_2 = (f_1 + f_2 - \sqrt{(f_1^2 + f_2^2)})\,(f_1^2 + f_2^2)^{m/2}$$

Note that R-functions can be used for describing many geometric primitives as well. For example, the description of a segment in E^1 can be obtained from the descriptions of two rays as follows:

$$f(x) = (x_1 - b_1) \,\&\, (b_2 - x_1)$$

The plot of this function for the expressions (3) is shown in Figure 1a with a=1, Figure 1b corresponds to (4) with a=0 and Figure 1c corresponds to (5) with m=1. It is important to point out that the function in Figure 1b and 1c does not have points in its domain where the derivative is discontinuous. Such points can cause problems in subsequent operations on objects, especially blending [8].

Blending set-theoretic operations

We propose to consider a blending surface as a boundary of an object obtained by the modified set-theoretic operations with the description based on the technique of R-functions. To explain our approach let us start from the two-dimensional example shown in Figure 2. Note that we use expressions (4) here. That is why f=C gives exact definition of the corner for C=0 and smooth contour lines for other C values.

The blending set-theoretic operation based on R-functions can be defined as

$$F(f_1,f_2) = R(f_1,f_2) + d(f_1,f_2) \qquad (6)$$

where R - is a corresponding R-function, d is a displacement function that has a maximal absolute value d(0,0) and asymptotically approximates a zero value with increasing absolute values of the arguments. We have found that the following simple form of the displacement function is suitable for blending:

$$d(f_1,f_2) = \frac{a_0}{1 + (f_1/a_1)^2 + (f_2/a_2)^2} \qquad (7)$$

The section $f_1=0$ for (7) is shown in Figure 3. The section $f_2=0$ has the same shape. It is assumed that the descriptive functions for both objects have the distance property. It is worthwhile to point out that the proposed displacement function is not the only one possible. Other Gaussian-like functions can be designed for specific applications.

Applying equations (4), (6) and (7) we can get, for example, the final description form of the blending intersection operation:

$$F(f_1,f_2) = f_1 + f_2 - \sqrt{(f_1^2 + f_2^2)} + \frac{a_0}{1 + (f_1/a_1)^2 + (f_2/a_2)^2}$$

This form has C^1 discontinuity only in points with $f_1=0$ and $f_2=0$. To get a C^m continuous function in the entire domain of definition one may apply the equations (5).

Parameters of the displacement function have clear meaning for shape control:

- The absolute value of a_0 defines the total displacement of the blending surface from the two initial surfaces.
- $a_0=0$ means pure set-theoretic operation.
- A negative a_0 value gives subtracted material blend, and a positive a_0 value yields added material blend.
- The values of $a_1 > 0$ and $a_2 > 0$ are proportional to the distance between the blending surface and the original surfaces defined by f_2 and f_1, respectively.

Figure 4 illustrates parameters interpretation for the intersection of two 2D halfspaces. The parameters have the same meaning for other set-theoretic operations.

Figure 5 illustrates an application of blending set-theoretic operations. The initial CSG object is shown in Figure 5a. The basic block is not defined as a single primitive but by intersecting six halfspaces to control the shape of edges in further blending. Figure 5b shows the same object with the blending set-theoretic operations applied. Different values of the displacement function parameters were assigned to the different edges. The prominent front edge of the block is defined by added material blending. Other edges were defined by subtracted material blending. Multiple blending operations were applied in the corners. The small cylindrical hole is made after blending to show that no problem occurs when applying set-theoretic operations to the blended object. This is ensured by the continuity of the function (6).

Aesthetic blending

Although the proposed blending operations provide easy shape control, they seem too indirect for aesthetic blending. Rather than change numeric parameters, a designer prefers to define the shape of an aesthetic blend with a single hand-drawn stroke. The simplest way is to show various planar sections of initial objects (or their projections) on a screen, and to input points of a user's drawn stroke. These points are then used to

a b

Figure 7. (a) an initial 2D solid; (b) a solid with the convex edges rounded by constant-radius blending.

estimate values of parameters that define blend as close to this stroke as possible. This is illustrated by Figure 6a with projections of two objects and the stroke shown. The stroke belongs to a certain plane. Points of hand-drawn strokes are assumed to belong to the blending surface. These constraints determine a system of nonlinear equations:

$$F(f_1(x_i,y_i,z_i), f_2(x_i,y_i,z_i), a_0,a_1,a_2) = 0, \quad i=1,N$$

where f_1 and f_2 are descriptive functions of initial objects, F is the blending set-theoretic operation (6) and N is the number of points. To find the best estimation of a_0, a_1 and a_2 in the sense of least squares, we applied a simple quadratic search using random points for the initial estimate. The resultant object for the example given in Figure 6a is shown in Figure 6b.

Constant-radius blending

If the parameters a_1 and a_2 of the displacement function (7) are equal to each other, the shape of blend looks like a sphere. Unfortunately, its shape also depends on the initial functions f_1 and f_2. That is why it is difficult to get a spherical blend with the defined radius. We suggest using another approach if exact constant-radius blending is needed. We follow [9] in applying offset operations to reach this goal. The continuity and distance property of the descriptive function of the object are also used here. The algorithm evaluating the descriptive function for positive offsetting with distance D is as follows:

1) define an initial object by $f(x,y,z) \exists 0$;
2) for any given point (x,y,z) define the sphere of radius D with the center in this point;
3) find maximal value f_0 of f(x',y',z') for points (x',y',z') belonging to the sphere surface;
4) assign $f(x,y,z)=f_0$;

The modification of this algorithm for negative offsetting can be done by finding the minimal function value in step 3. The procedure for rounding convex edges of the object includes consecutively applied negative and positive offsetting. Figure 7 illustrates this procedure for the functionally represented 2D solid.

Conclusion

In this paper, we have proposed an exact analytical definition of the blending set-theoretic operations over geometric solids with implicitly defined surfaces. By using R-functions and introducing the displacement function we obtained a continuous descriptive function with easy control of the blending surface shape. Subtracted and added material blends are supported for all kinds of set-theoretic operations. The resultant solid can be treated in the same manner as the initial solid. Thus, both traditional and blending set-theoretic operations can be applied to it again.

The desired shape of an aesthetic blend can be defined by a hand drawn stroke. The stroke imposes constraints on blend parameters that can be satisfied by a solution of a system of nonlinear equations. We plan to deal with functional constraints using the same technique. A procedure of evaluating a descriptive function for constant-radius blending based on the offsetting operation was proposed.

This work is part of our project to develop a rich system of operations for the function representation (or F-rep) of multidimensional geometric objects [5]. Some other operations already investigated are bijective mapping, projection, morphing, sweeping and hypertexturing. All these operations can be applied to an object resulting from the blending operations.

Acknowledgements

Authors would like to thank Dr. V.Adzhiev from the University of Warwick (UK) and Dr. A.Sourin from the Nanyang Technological University (Singapore) for fruitful discussions. Our sincere thanks to John Sarraille for his comments on stylistic issues of the manuscript.

References

1. C. Hoffmann and J. Hopcroft, "The potential method for blending surfaces and corners", Geometric Modeling: Algorithms and New Trends, G.Farin, Ed., SIAM, Philadelphia, (347-365), 1987.

2. L. Holmstrom, "Piecewise quadric blending of implicitly defined surfaces", Computer Aided Geometric Design 4, 3 (171-189), 1987.

3. T. Lowe, M. Bloor and M. Wilson, "Functionality in blend design", Computer Aided Design 22, 10 (655-665), 1990.

4. A.E. Middleditch and K.H. Sears, "Blend surfaces for set theoretic volume modeling systems", Computer Graphics 19, 3 (161-170), 1985.

5. A. Pasko, V. Savchenko, V. Adzhiev and A. Sourin, "Multidimensional geometric modeling and visualization based on the function representation of objects", Technical Report 93-1-008, The University of Aizu, Japan, 1993.

6. A. Ricci, "A constructive geometry for computer graphics", The Computer Journal 16, 2(157-160), 1973.

7. A. Rockwood and J. Owen, "Blending surfaces in solid modeling", Geometric Modeling: Algorithms and New Trends, G. Farin, Ed., SIAM, Philadelphia, (367-383), 1987.

8. A.P. Rockwood, "The displacement method for implicit blending surfaces in solid models", ACM Transactions on Graphics 8, 4(279-297), 1989.

9. J.R. Rossignac and A.A.G. Requicha, "Constant-radius blending in solid modeling", Computers in Mechanical Engineering 3, 1(65-73), 1984.

10. V.L. Rvachev, "On the analytical description of some geometric objects", Reports of Ukrainian Academy of Sciences 153, 4 (765-767), 1963.

11. V.L. Rvachev, Methods of Logic Algebra in Mathematical Physics, Naukova Dumka Publishers, Kiev, 1974.

12. J.Warren, "Blending algebraic surfaces", ACM Transactions on Graphics 8, 4 (263-278), 1989.