Spectral H_2 Synthesis of Fault Estimation Observer

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ABSTRACT
This paper is devoted to the novel 2-step approach to construction of the fault estimation observer. First, we propose to design the auxiliary fault detection observer, and then to use this one for the initial problem solution. Special spectral algorithm of SISO (Single Input and Single Output) H_2 optimization is implemented in the range of proposed approach. Simulation results are presented to demonstrate applicability and effectiveness of proposed techniques.

Categories and Subject Descriptors
I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search –*control theory*

General Terms
Algorithms.

Keywords
fault detection, fault estimation, control system, optimization, H_2-control, feedback, functional.

1. INTRODUCTION
Fault detection and estimation problems have received serious attention in scientific publications during last two decades (some significant papers are cited in [1-3]). On the other hand, such problems with the initially known spectral features of external disturbances are not explored nowadays.

The matter of this research is to design an adaptive observer for detection and estimation of constant (or slow-varying) additive faults. The technique proposed in this paper is inspired by the special spectral approach to SISO H_2 mean-square optimal synthesis problem [4].

This paper is organized as follows. In Section 2, we present the description and statement of the problem. Sections 3 and 4 describe a design of the auxiliary fault detection observer (the first step) and the required fault estimation one (final step). In Section 5, one numerical example is given to illustrate an implementation of the proposed approach. Section 6 concludes the paper by overall resume.

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2. PROBLEM STATEMENT
Let us consider the following LTI system

$$\dot{x} = \mathbf{A}x + \mathbf{b}u + \mathbf{E}f + \mathbf{P}d\,, \quad y = \mathbf{C}x\,, \tag{1}$$

where $x \in \mathbf{R}^n$ is the vector of the system state, *y*, *u*, *f*, *d* are the following scalars: the output signal, the control one, additive fault action and the external disturbance with the known spectral power density. Let us suppose that the value of the fault derivative may be neglected, i.e. $\dot{f}(t) \approx 0$.

Adaptive fault estimation observer-filter, close to the variant described in [5], has the following structure:

$$\begin{aligned} &\dot{\hat{x}} = \mathbf{A}\hat{x} + \mathbf{b}u + \mathbf{E}\hat{f} + \mathbf{L}_x \nu_1\,, \\ &\dot{\hat{f}} = \nu_2\,, \nu_1(s) = V(s)(y - \mathbf{C}\hat{x})\,, \\ &\nu_2(s) = W(s)(y - \mathbf{C}\hat{x})\,, \end{aligned} \tag{2}$$

where ν_1, ν_2 are corrective terms, vector $\mathbf{L}_x$, and transfer functions $V(s) = V_1(s)/V_2(s)$, $W(s) = W_1(s)/W_2(s)$ are to be computed. Designed observer-filter (2) should generate fast and accurate fault estimation signal despite of presence of external disturbances.

Let us denote

$$e_x = x - \hat{x}\,,\ e_y = y - \mathbf{C}\hat{x}\,,\ e_f = f - \hat{f}\ , \tag{3}$$

describe an error dynamics of the observation by the model of the form

$$\begin{aligned} &\dot{e}_x = \mathbf{A}e_x + Ee_f - \mathbf{L}_x \nu_1\,, \\ &\dot{e}_f = -\nu_2\,, \nu_1(s) = V(s)e_y\,, \\ &\nu_2(s) = W(s)e_y\,. \end{aligned} \tag{4}$$

Rewrite this model in frequency domain:

$$sA(s)V_2(s)e_y(s) = -sL(s)V_1(s) - E(s)V_2(s)\nu_2(s) + {} + sP(s)V_2(s)d(s)\text{, where} \tag{5}$$

$$\begin{aligned} &A(s) = \det(\mathbf{I}s - \mathbf{A}), E(s) = A(s)\mathbf{C}(\mathbf{I}s - \mathbf{A})^{-1}\mathbf{E}, \\ &P(s) = A(s)\mathbf{C}(\mathbf{I}s - \mathbf{A})^{-1}\mathbf{P}, L(s) = A(s)\mathbf{C}(\mathbf{I}s - \mathbf{A})^{-1}\mathbf{L}_x\,, \end{aligned} \tag{6}$$

and consider the transfer function from $d(t)$ to $e_f(t)$

$$e_f(s) = F_{e_f d}(s)d(s) = \frac{P(s)V_2(s)W_1(s)}{\Delta_2(s)} d(s), \text{ where} \quad (7)$$

$$\Delta_2(s) = sA(s)V_2(s)W_2(s) + sL(s)V_1(s)W_2(s) + E(s)V_2(s)W_1(s). \quad (8)$$

We suppose that the mentioned disturbances have the following initially given spectral power density

$$S_d(s) \equiv S_1(s)S_1(-s), \ S_1(s) \equiv N_d(s)/T(s), \text{ where}$$

$$S_1(s) \equiv \frac{N_d(s)}{T(s)} = \sqrt{\frac{4D_r\alpha}{\pi}} \frac{1}{s^2 + 2\alpha s + \alpha^s + \beta^2}, \quad (9)$$

where β is the central frequency, and $\alpha = s_t\beta$, where s_t is the blurriness. Let note that constant factor can be ignored for the SISO case in a range of design algorithms (i.e. $N_d(s) \equiv 1$ in this paper). To simplify disturbance representation, we can also use its polyharmonical form as follows:

$$d(t) = \sum_{i=1}^{n_h} A_{di}\sin(\sigma_i t + \varphi_i) \quad (10)$$

where A_{di}, σ_i, φ_i are amplitudes, frequencies and phases of the corresponding harmonics. The influence of $d(t)$ to the fault estimation process can be expressed by the value

$$J_\omega = \max_i \{A_{di}|F_{e_f d}(j\sigma_i)|\}. \quad (11)$$

Remark that this value and the value of the fault estimation process settling time T_p are functions of $\mathbf{L}_x$, $V(s)$, and $W(s)$. In such a way, the problem to be considered is to design the mentioned items of observer such that

$$J_\omega(L, V(s), W(s)) \le J_\omega^0, \ T_p(L, V(s), W(s)) \le T_p^0, \quad (12)$$

where J_ω^0, T_p^0 are given desired values of J_ω and T_p.

3. FAULT DETECTION

First, let us design the fault detection observer-filter, which has the only task: to detect the presence of the fault. The correspondent equations of the filter are as follows:

$$\dot{\hat{x}} = \mathbf{A}\hat{x} + \mathbf{b}u + \mathbf{L}_x\nu_1, \quad \nu_1(s) = V(s)(y - \mathbf{C}\hat{x}), r = e_y = (y - C\hat{x}), \quad (13)$$

where scalar r is the residual signal, and the rest variables are described above. The residual signal must be sensitive to the constsnt fault f and senseless with respect to periodic disturbance d. Note that the adaptive observer-filter (13) is the basis for a solution of the main problem. Let us describe an error dynamics of the observation (1) by the model (13) in frequency domain of the form

$$A(s)e_y(s) = -L(s)\nu_1 + (P(s)d(s) + E(s)f(s)), \quad (14)$$

using the notations (3), (6). We can reduce (14) to the form

$$A(s)e_y(s) = -\tilde{\nu}_1 + (P(s)d(s) + E(s)f(s)), \quad \tilde{\nu}_1 = \tilde{V}(s)e_y(s) = \frac{\tilde{V}_1(s)}{V_2(s)} e_y(s) = L(s)V(s)e_y(s), \quad (15)$$

using new corrective term $\tilde{\nu}_1(s)$. Let us consider transfer functions $F_{e_y d}(s), F_{e_y f}(s)$ from d and f to e_y $F_{e_y d}(s) = P(s)V_2(s)/\Delta_1(s)$, $F_{e_y f}(s) = E(s)V_2(s)/\Delta_1(s)$, where $\Delta_1(s)$ is the characteristic polynomial of the closed-loop connection (15)

$$\Delta_1(s) = A(s)V_2(s) + \tilde{V}_1(s). \quad (16)$$

Transfer function $\tilde{V}(s) = \tilde{V}_1(s)/V_2(s)$ should be chosen to maximize the functional

$$J_1(\tilde{V}) = J_1^1(\tilde{V}) / J_1^2(\tilde{V}), \text{ where} \quad (17)$$

$$J_1^1(\tilde{V}) = |F_{e_y f}(0)|, \ J_1^2(\tilde{V}) = |F_{e_y d}(\beta_0)|.$$

The given statement of the proposed problem is close to well-known H_-/H_∞ optimization approach [1]. Remark that this problem of simultaneous search of the control action vector and the optimal transfer function has an analytical solution.

One can easy see that the frequency response $|F_{e_y d}(j\omega)|$ has a dip in the neighborhood of β_0 to minimize $J_1^2(\tilde{V})$, i.e. $V_2(j\beta_0) \approx 0$. As a result, obtain

$$V_2(s) = \tilde{V}_2(s)\bar{V}_2(s, \beta_0), \ \bar{V}_2(s, \beta_0) = (s^2 + 2\varepsilon s + \beta_0^2), \quad (18)$$

where ε is a positive constant value close to zero. Let us consider the problem of $J_1^1(\tilde{V})$ maximization. The theorem of the root distribution [6] states that any polynomial $\Delta(s)$, $deg\Delta(s) = m_d$ with the degree of stability $\alpha_{st} > 0$ has the corresponding vector $\gamma \in R^{m_d}$, such as $\Delta(s) \equiv \Delta^*(s, \gamma)$, where

$$\Delta^*(s, \gamma) = \begin{cases} \tilde{\Delta}^*(s, \gamma), \\ (s + a_{d+1}(\gamma, \alpha_{st}))\tilde{\Delta}^*(s, \gamma), \end{cases} \quad (19)$$

$$\tilde{\Delta}^*(s, \gamma) = \prod_{i=1}^{d} (s^2 + a_i^1(\gamma, \alpha_{st})s + a_i^0(\gamma, \alpha_{st})),$$
$$a_i^1(\gamma, \alpha_{st}) = 2\alpha_{st} + \gamma_{i1}^2, a_i^0(\gamma, \alpha_{st}) = \alpha_{st}^2 + \gamma_{i1}^2\alpha_{st} + \gamma_{i2}^2, \quad (20)$$
$$a_{d+1}(\gamma, \alpha_{st}) = \gamma_{d0}^2 + \alpha_{st}, d = [m_d/2],$$
$$\gamma = \{\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}, ..., \gamma_{d1}, \gamma_{d2}, \gamma_{d0}\}.$$

There exists the relationship between the degree of stability and sensitivity to constant disturbance action (which should be maximized). Note that the value $|F_{e_y f}(0)|$ is inversely proportional to $\Delta_1(0)$ and let define $\deg\Delta_1(s) = n_1$ and its degree of stability as α_{st}. In accordance to the mentioned theorem, there exists γ^*, such as $\Delta_1(s) = \Delta_1^*(s, \gamma)$, constructed with formulas (19), (20), and

$$\Delta_1^*(0, \gamma^*) = \prod_{i=1}^{n_1} a_i^0(\gamma^*, \alpha_{st}) = \prod_{i=1}^{n_1} (\alpha_{st}^2 + \gamma_{i1}^{*2}\alpha_{st} + \gamma_{i2}^{*2}) \ge \alpha_{st}^{2n_{11}}.$$

This means that the value $|F_{e_y f}(0)|$ is inversely proportional to α_{st}. We use modal parametric synthesis approach, allowing to set the degree of stability for the polynomial (16) initially. Degree of the polynomial $L(s)$ is taken equal to $n-1$. Let formulate the algorithm of the observer (13) design.

Algorithm 1:

1. Define degrees of the polynomials $\tilde{V}_1(s)$, $V_2(s)$ in (18) and therefore $n_1 = deg\Delta_1(s)$, set α_{st}, $\gamma_1^0 \in R^{n_1}$, compute $\Delta_1^*(s,\gamma_1^0)$ by formulas (19), (20).

2. Solve the polynomial equation

$$A(s)V_2(s) + \tilde{V}_1(s) = \Delta_1^*(s,\gamma_1^0).$$

3. Choose any $n-1$ roots ξ_i, $i=\overline{1,n-1}$ of the polynomial $\tilde{V}_1(s)$, then calculate $L(s)=\prod_{i=1}^{n-1}(s-\xi_i)$, corresponding $\mathbf{L}_x$, such that $\mathbf{C}(\mathbf{I}s-\mathbf{A})^{-1}\mathbf{L}_x = L(s)$, and $V_1(s)=\tilde{V}_1(s)/L(s)$. Then evaluate $J_1(\tilde{V})$ by (17).

4. Maximize $J_1(\tilde{V})$ (17), repeating steps 2-4, using any numerical method. Use optimal $\gamma_1=\gamma_1^*$ to compute required items $V_0(s)=V_1^0(s)/V_2^0(s)$ and $\mathbf{L}_{x0}$.

4. FAULT ESTIMATION

The object of this section is to construct the fault estimation observer (2), using the vector $\mathbf{L}_{x0}$, and the transfer matrix $V_0(s)=V_1^0(s)/V_2^0(s)$, computed above. Now let rewrite the expression (5) of the model (4):

$$A_1(s)e_y(s) = E_1(s)\nu_2(s) + P_1(s)d(s), \text{ where}$$

$$A_1(s) = sA(s)V_2^0(s) + sL_0(s)V_1^0(s), L_0(s) = \mathbf{C}(\mathbf{I}s-\mathbf{A})^{-1}\mathbf{L}_{x0},$$
$$P_1(s) = sP(s)V_2^0(s)(s), E_1(s) = -E(s)V_2^0(s).$$

Note that spectral density of d is presented by (9). The problem (7) of sensitivity minimization for the disturbance d with the central frequency β_0 can be formulated as the following mean-square optimization problem:

$$\tilde{J} = \tilde{J}(W) = \lim_{T\to\infty}\frac{1}{T}\int_0^T (e_y^2 + k^2\nu_2^2)dt \to \min_W,$$

where the parameter k characterizes a balance between the sensitivity and stability degree of the system. If we present d in the form (9), the special approach to H_2 optimization, described in papers [4, 5], can be used to solve this problem. Nevertheless, there is one serious trouble: the polynomial $N_2(s)$, which is determined by the formula

$$N_2(s)N_2(-s) = -s^2P(s)P(-s)V_2^0(s)V_2^0(-s)\times \times N_d(s)N_d(-s), \quad (21)$$

used in the following calculations, is submultiple for the characteristic polynomial (8) of the system closed by the optimal $W(s)$, computed in accordance with [4]. One can easy see that to provide the asymptotical stability, the factor s in (21) must be changed to $(s+p)$, where $p>0$ is a small number, i.e. N_2 must be changed to $\tilde{N}_2(s)$ as follows:

$$\tilde{N}_2(s)\tilde{N}_2(-s) = (s+p)(-s+p)P(s)P(-s)\times \times V_2(s)V_2(-s)N_d(s)N_d(-s). \quad (22)$$

Let us also note, that the parameter α in the (9) should be set close to zero to save frequency properties.

It is necessary to pay attention to the fact, mentioned in paper [5], that computed transfer function may be improper that obstructs its practical realization. One of ways to avoid this difficulty, is to deform the spectral power density (22) $\tilde{N}_2^*(s)=\tilde{N}_2(s)N^*(s)$, where $N^*(s)$ is a Hurwitz polynomial of the degree n^* by the way pointed in [5]. To save degree of stability of the closed-loop system, we accept it equal to p for $N^*(s)$ and parameterize $N^*(s)$ as $N^*(s)=N^*(s,\gamma_2)$, where $\gamma_2 \in R^{n^*}$, using (19), (20). As a result, we have

$$\tilde{N}_2^*(s) = \tilde{N}_2^*(s,\gamma_2) = \tilde{N}_2N^*(s,\gamma_2).$$

Process of $W(s)$ computation consists of the following steps:

1. Execute the factorization of the polynomial:

$$k^2A_1(s)A_1(-s) + E_1(s)E_1(-s) \equiv G(s)G(-s), \quad (23)$$

2. Construct the auxiliary polynomial

$$R(s) = \sum_{i=1}^{n}\frac{G(-s)}{g_i-s}\frac{E_1(-g_i)\tilde{N}_2^*(g_i)}{A_1(g_i)T(g_i)G'(-g_i)}, \quad (24)$$

where g_i, $i=\overline{1,n}$ are the distinct roots of $G(-s)$.

3. Represent a transfer function of the optimal filter

$$W(s) = \frac{[A_1(s)T(s)R(s)+E_1(-s)\tilde{N}_2^*(s)]/G(-s)}{[E_1(s)T(s)R(s)-k^2A_1(-s)\tilde{N}_2^*(s)]/G(-s)}, \quad (25)$$

where division to polynomial $G(-s)$ is done totally.

The optimal $W(s)$ is a function of the parameters γ_2, k, p. As a result, solution of the problem (12) is equal to minimization of the following functional

$$J_2(W) = J_2(\gamma_2,k,p) = T_p - T_p^0 + |T_p - T_p^0| + \ldots \ldots + J_p - J_p^0 + |J_p - J_p^0| + J_\omega - J_\omega^0 + |J_\omega - J_\omega^0|, \quad (26)$$

where $J_\omega = J_\omega(\gamma_2,k,p)$ is described by (11), $T_p = T_p(\gamma_2,k,p)$ is the time of fault estimation, and $J_p = J_p(\gamma_2,k,p)$ is the overshoot. Optimal parameters $k=k^*$, $p=p^*$ and $\gamma_2=\gamma_2^*$ can be designed with the following algorithm.

Algorithm 2.

1. Set initial $k = k_0$, $p = p_0$ and vector $\gamma_2 = \gamma_2^0 \in R^{n^*}$. Compute the polynomial $\tilde{N}_2^*(s) = \tilde{N}_2^*(s, \gamma_2)$.

2. Compute the function $W(s)$, using formulas (23-25).

3. Evaluate $J_2(\gamma_2, k, p)$ (26).

4. Obtain vector $\gamma_2 = \gamma_2^*$, minimizing $J_2(\gamma_2^*, k, p)$ (26), using any numerical method, e.g. Nelder-Mead algorithm. If $J_2(\gamma_2^*, k, p)$ is not close to zero, then repeat steps 2-4 with new parameters k, p, searched, e.g., with enumeration.

5. Compute optimal $W_0(s) = W_1^0(s) / W_2^0(s)$.

5. NUMERICAL EXAMPLE

Consider the model (1) of the yaw ship motion with the constant speed, consisting of the following matrices

$$\mathbf{A} = \begin{pmatrix} -0.0936 & 0.634 & 0 \\ 0.048 & -0.0717 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \ \mathbf{b} = \begin{pmatrix} 0.0196 \\ 0.0160 \\ 0 \end{pmatrix}, \ \mathbf{E} = \mathbf{b},$$

$$\mathbf{P} = (0.41 \quad 0.0076 \quad 0)^T, \ \mathbf{c} = (0 \quad 0 \quad 1),$$

and the external disturbance (10) with $\beta_0 = 0.45$:

$$d(t) = 0.1 sin(0.9\beta_0 t) + 1 sin(\beta_0 t) + 0.1 sin(1.1\beta_0 t).$$

Firstly, we compute parameters of the fault detection observer (13), where $V_1(s)$, $V_2(s)$ are polynomials of second degree $V_2(s) = \overline{V}_2(s) = s^2 + 0.002s + 0.2025, \tilde{V}_2(s) = 1$. Setting $\alpha_{st} = 0.03$, $\gamma_1^0 = (11111)$ we use the Algorithm 1 and obtain

$$V_0(s) = \frac{V_1^0(s)}{V_2^0(s)} = \frac{4.46 + 1.99s + 0.28}{s^2 + 0.002s + 0.2025},$$

$$\mathbf{L}_{\mathbf{x0}} = (-0.87 \quad 0.46 \quad 1)^T.$$

The following parameters of the fault estimation observer are computed in accordance with the Algorithm 2. We accept $n^* = 3$ and use the initial parameter vector $\gamma_2^0 = (111)$. As a result, the optimal terms are the following:

$$k^* = 0.1, \ p^* = 0.02, \ N^*(s) = s^3 + 1.79s^2 + 2.15s + 1.45,$$

$$W_1^0(s) = 99.09\,s^7 + 542\,s^6 + 843.9\,s^5 + 449.3\,s^4 +$$
$$+ 212.2\,s^3 + 68.49\,s^2 + 8.788\,s + 0.158,$$

$$W_2^0(s) = s^7 + 4.646\,s^6 + 7.948\,s^5 + 7.858\,s^4 + 6.512\,s^3$$
$$+ 1.863\,s^2 + 0.9995s + 0.09226.$$

Figures 1, 2 illustrate frequency response $A(\omega)$ of the $F_{e_f d}(s)$ (7) for the designed observer and the fault estimation process. The dip in the area of the central frequency β_0 can be seen in the Fig. 1. $A(\omega)$ can be compared with the response $A_0(\omega)$ of the observer (3) with the constant parameters $\mathbf{L}_{\mathbf{x}} = (36.0 \quad 23.0 \quad 6.8)^T$, $V(s) \equiv 1$, $W(s) \equiv 3162.3$ and structure described in [2,3].

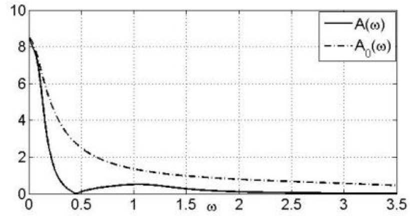

Figure 1. Frequency responses $\mathbf{A}(\omega)$ and $\mathbf{A_0}(\omega)$.

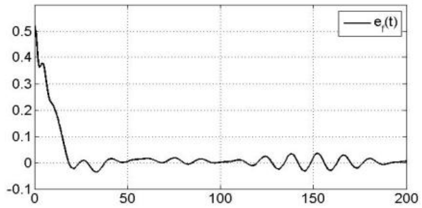

Figure 2. Fault estimation process (error dynamics).

6. CONCLUSION

In this paper, a novel approach to the fault estimation has been presented. The simulated result demonstrates its applicability with respect to the external disturbance with the given frequency range. Sincerely, proposed approach has a serious disadvantage: it cannot be applied to rapidly varying fault estimation. The object of the future research is spectral solution of such faults estimation, maybe taking into account time delays or robust features.

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