Forecasting the Dynamics of Key Macroeconomic Indicators Based on Simulation

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ABSTRACT
In this paper, we propose and explore a model for forecasting the development of the economy, formalized as the Cauchy problem for a system of stochastic differential equations taking into account "turning points". For the numerical solution of the corresponding computational problem, the Euler-Maruyama method is used. Stochastic simulation for different "what if" scenarios and several countries is performed using MATLAB.

Categories and Subject Descriptors
I.6.3. [**Simulation and modeling**]: Applications
J.4. [**Social and Behavioral Sciences**]: Economics

General Terms
Algorithms, Economics, Experimentation

Keywords
Macroeconomic indicators, dynamic models, Euler-Maruyama method, stochastic simulation, MATLAB

1. INTRODUCTION
The modern economy as it develops gains new properties and characteristics that make for a fresh look at the problems of forecasting economic dynamics including economic changes and crises. The main of them are connected with the expansion of globalization impact on the macroeconomic situation in separate countries and regions, the effect being significantly intensified due to the development of innovative processes. Models and methods designed nowadays to describe and forecast in macroeconomics, often rely heavily on the use of econometric models [3], [4]. This approach, based on preceding trends and patterns, requires large data sets containing macroeconomic indicators that are usually collected from different databases with respect to a long period of time. This greatly complicates the construction of reliable forecasts of economic development, taking into account the modern economic conditions. First, though econometric models use a large number of explanatory variables they are not free of missing essential macroeconomic variables or being based on the wrong variables. Secondly, in theory they rely on datasets where the time dimension is fairly long. But for many important data panels (e.g., for the Euro area or for the transition countries) the number of observations hardly exceeds 25-30.

An alternative is to consider dynamic models based on the idea that developments in the data are driven by only a few key factors [1], [6]. Provided these factors are identified the approach results in a small dimension model that may be used to produce "what if" scenarios and generate economic forecasts. Of particular importance for describing the development of the modern economy with the use of such models is the possibility of taking into account the random processes liable to influence the key factors, in particular, in the framework of stochastic dynamic general equilibrium models (SDGE).

Following [4] we make the difference between the terms "prediction" and "forecast" though they are interchangeable in many fields. The difference is not only etymological. We treat a prediction as a definitive and specific statement about the value of the indicator under consideration. Whereas a forecast is a probabilistic statement, usually over a long time scale.

We propose a dynamic model for forecasting that describes the changes in key macroeconomic indicators, taking into account only the current state of the economy. The model is described in Section 2. The discrete model underlying the simulation is shown in Section 3. The results of numerical experiments are illustrated in Section 4 with GDP dynamics for Finland and Japan. Section 5 concludes.

2. PROBLEM FORMULATION
Constructing the model, we took into account the following assumptions:
a) the conditions of the economic growth model for a closed economy are satisfied;
b) production function and technological changes are described in the form of an AK-model;
c) investment are instantly converted into capital (no lag).

Given these assumptions, the forecasting model has the form of the Cauchy problem for the system of stochastic differential equations (SDE) (see [5] for details):

$$\begin{cases} dY(t) = (A(t) - \delta)K(t)dt + A(t)K(t)dW(t) \\ dK(t) = ((A(t) - \delta)K(t) - C(t))dt + K(t)dW(t) \\ dC(t) = A(t)K(t)dt - (1-\delta)K(t)dt + A(t)K(t)dW(t) \end{cases} \quad (1)$$

with initial conditions $Y(0) = Y0$, $K(0) = K0$, $C(0) = C0$. $Y(t)$, $K(t)$ and $C(t)$ in (1) stand for gross output, capital stock and consumption at time t, $dY(t)$, $dK(t)$ and $dC(t)$ denote the increment of gross output, capital stock and consumption for the period $[t, t+dt]$, $A(t)$ denotes technological factor at t, δ is the capital depreciation rate. $dW(t)$ stands for the increment of the one-dimensional Wiener process (with the zero expectation and variance $\sigma_k^2 dt$).

3. DISCRETE APPROXIMATION

To construct a computable algorithm for solving the SDE system (1) we use the method for the approximate numerical solution of a stochastic SDE known as the Euler–Maruyama method [2]. It is a simple generalization of the Euler method for deterministic ordinary differential equations. The Euler–Maruyama approximation to the true solution is the Markov chain. The discrete approximation of (1) over $[0, T]$ and the time step Δ, provided the random variable dW is considered as discretized Brownian motion ($W(t)$ is specified at discrete t values), takes the form

$$\begin{cases} Y_{t+\Delta} = Y_t + (A_t - \delta_t)K_t\Delta + A_t K_t \sigma_k \sqrt{\Delta}\xi_t \\ K_{t+\Delta} = K_t + ((A_t - \delta_t)K_t - C_t)\Delta + K_t \sigma_k \sqrt{\Delta}\xi_t \\ C_{t+\Delta} = C_t + A_t K_t \Delta - (1+\delta_t)K_t\Delta + A_t K_t \sigma_k \sqrt{\Delta}\xi_t \end{cases} , \quad (2)$$

where ξ_t are independent and identically distributed normal random variables with expected value zero and variance 1.

Note that for computational purposes the choice of the step size requires taking into account two considerations. First, the order of convergence for the Euler-Maruyama method is known to be low (strong order of convergence 0.5), so the results of calculations are inaccurate unless a small enough step size is used [5]. Secondly, the step should be chosen large enough to leave stochastic component meaningful.

4. SIMULATION

4.1 Software environment

When choosing the software environment for simulation, we were guided by the fact that MATLAB is an ideal one for generating and treating artificial data because of its

a) vectorizing core principle;
b) high-quality random number generators;
c) the opportunity to make experiments repeatable;
d) excellent graphics facilities;

Besides, an open architecture of MATLAB allows to modify any existing routine or to include new.

4.2 Simulation scenarios

We have performed a detailed three-part simulation study using data generation processes based on (2). In the first series of numerical experiments, we simulated the values of GDP, capital stock and consumption for 10 countries in different time periods and with different initial conditions. Master data for initialization the simulation were taken from statistical sources[1]. For parameters that are not available directly in the statistics, the procedure of model calibration has been applied, based on comparison of the simulation results with statistical data.

The results of the simulation showed that the model we proposed describes quite well the dynamics of key macroeconomic parameters for a short period of time (3-4 years), provided that there are no "turning points" - abrupt changes in the state of the economy.

Simulations of the second type were designed to account for the "turning points". To do this we used the technique of introduction of a dummy variable, a numeric stand-in for some qualitative fact (often used in time series analysis with regime switching). The appointment of dummy variable is to change abruptly the value of the capital depreciation rate δ at the particular time moment.

In the third part of the simulation study we performed "what if" scenarios for every of 10 countries: the baseline (inertial) one and the alternative one, taking into account the "shock" changes in the economy state at the "turning points". The baseline scenario assumes that changes occur to the extent that they could have been foreseen (no "turning points"). It corresponds to the constant value of depreciation rate δ during all the time period. The alternative case means short-term and medium-term effects of macroeconomic "shocks" caused by the external shocks (e.g. a fall in world oil prices, sharp fluctuations in world demand for exported goods and services, interest rates and exchange rates); the shocks due to the tariff policy of natural monopolies; fiscal shocks; the investment policy; reserve and monetary policy shocks; social policy shocks, etc. Determining the precise date of a "turning point" and the value of dummy variable that changes δ requires some time after the event has passed so simulation is the effective tool to for the analyzing "back" and determine the parameters that characterized previous periods or forecasting "forward" the dynamics of the economy state.

Countries we have chosen for the simulation experiments, varied considerably in the level and rate of economic development. The simulation results are illustrated below with the examples of GDP trajectories for two countries – Finland and Japan. We deliberately have chosen completely different examples with "turning points", caused by different economic reasons, to test the simulation model.

The greatest difficulties in applying the model are related to its calibration, which includes both the definition of the parameters that are not available in the statistics and the detection of "turning points" – those time points when the trend of the macroeconomic indicators development changes sharply. When carrying out simulation experiments for Finland and Japan we determined numerically the value of parameter δ in the equations (1), (2) and the value of the dummy variable responsible for its leap in "turning points", so as to ensure the conformity of the 50 % confidence domain for the mean of the

[1] http://www.be5.biz/makroekonomika/, http://www.ereport.ru/

simulated random process and the actual trajectories of the macroeconomic indicators. The exact date of each "turning point" was determined from the analysis of statistical data on macroeconomic indicators. Provided the necessary information is available, other approaches to the model calibration may be used, for example, based on statistical methods for the analysis of theoretical and calculated indicator trajectories compliance.

4.3 Macroeconomic indicators for Finland

Finland is traditionally characterized with a high level of income and well-being. The "turning points" in economy of Finland in 2004-2016 are connected with two events: the global downturn and the beginning of the war of sanctions and counter-sanctions that have dragged down the global output.

The Figure 1 shows the simulated GDP trajectories (thin dotted line), the average and the borders of the 50% confidence domain (bold dotted line), and statistic data (bold points).

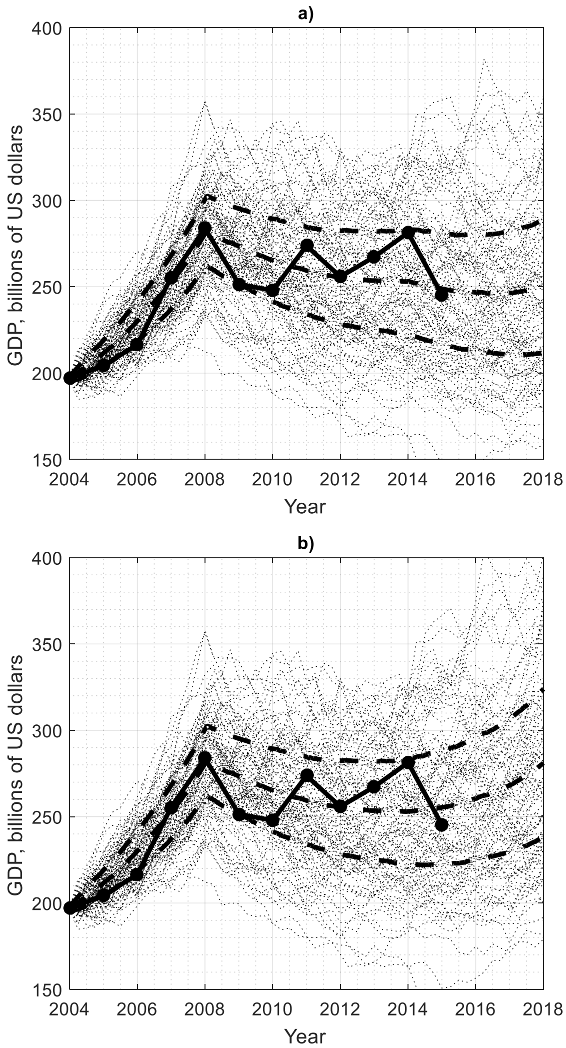

Figure 1. The GDP trajectories for the Finnish economy in 2004-2018: a) – two "turning points", b) - one "turning point".

The (a) subplot shows the dynamics of GDP trajectories simulated for a scenario that takes into account both turning points, the (b) subplot relates to the development of the process under the assumption that the second turning point is missing.

4.4 Macroeconomic indicators for Japan

The Japanese economy is one of the most developed economies in the world. The "turning points" for the economy of Japan in 2005-2016 are due to two events: the Global Economic Crisis that was not of great impact, and the unprecedented Triple Disaster 2011 that caused the immediate severe damage to Japan’s economy.

The Figure 2 shows the simulated GDP trajectories (thin dotted line), the average and the borders of the 50% confidence domain (bold dotted line), and 2005-2015 statistic data (bold points). The (a) subplot corresponds to the "two turning points" scenario, (b) – to the "one turning point" variant.

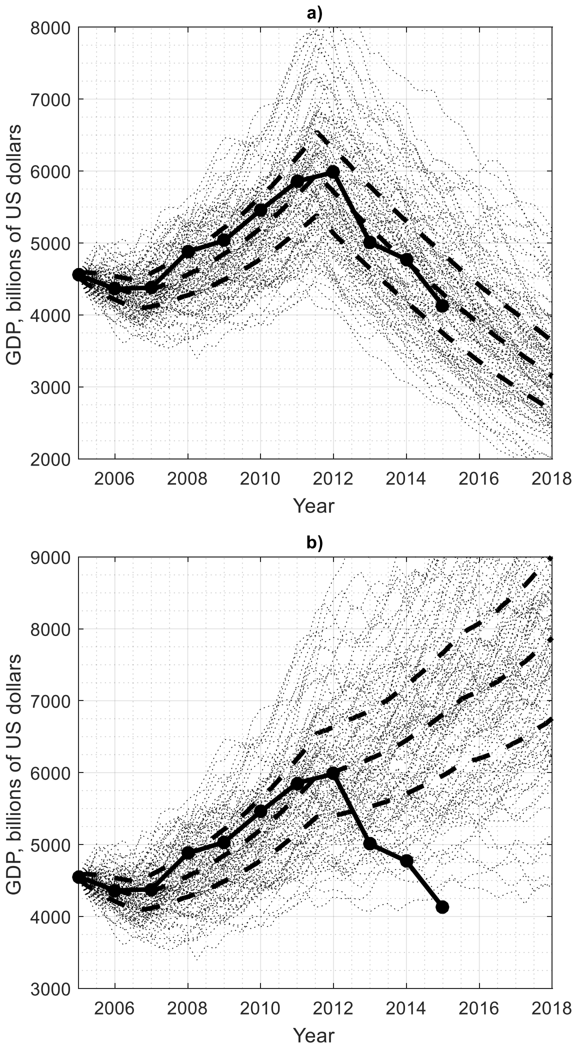

Figure 2. The GDP trajectories for the Japanese economy in 2005-2018: a) – two "turning points", b) - one "turning point".

5. CONCLUSIONS

The development of globalization and the increasing influence of uncertainty factors require methods to make forecasts based on the current state of the economy. In this paper, we proposed a computationally simple approach for forecasting the dynamics of key macroeconomic indicators. The forecasting model has the form of the Cauchy problem for the system of stochastic differential equations and its discrete approximation based on the algorithm of the Euler-Maruyama method. It allows arranging simulation study taking into account various scenarios including "turning points".

The advantages of the proposed approach are the possibility to quickly adjust the forecasts at the first sign of a changing macroeconomic trends, and the ability to solve the inverse problem, i.e., identify the model parameters that are not available in the statistics by comparing the simulation results and the statistical data.

The main problems encountered in the application of the approach, are connected with the model calibration, the justification of the economic information for setting initial conditions, determining the date of a "turning point" and the corresponding value of the dummy variable.

Further development of the forecasting model can be associated with the use of an improved algorithm of Euler-Maruyama, based on the use of more sophisticated approximations of stochastic differential equations.

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