# Multiprogrammed Control Problem of One Supply Chain

Inna Trofimova
St. Petersburg State University,
7/9, Universitetskaya Embankment,
St.Petersburg, Russian Federation, 199034 and
St.Petersburg institute for informatics and
automation of the Russian academy of sciences
39, 14 Line, St.Petersburg, 199178, Russia
isolovyeva@mail.ru

Larisa Mynzu
St. Petersburg State University,
7/9, Universitetskaya Embankment,
St.Petersburg, Russian Federation, 199034
larisabarisa@gmail.com

## ABSTRACT

In this paper the problem of construction of the multiprogrammed control for desired (opportune) programmed regimes is considered. To demonstrate proposed methods, we applied them to the model of one supply chain. Results are verified with the help of this procedure realization on numerical examples.

## Categories and Subject Descriptors

PBUH [**Optimization**]: Linear Programming; PBWH [**Mathematics**]: Mathematical Modelling

## General Terms

Mathematical model of supply chain, control problem

## Keywords

Multiprogrammed control, positional optimization method, supply chain

## 1. INTRODUCTION

Nowadays growth of global market causes progressive improvement of operating in modern supply chains (SC). Accomplish this SC strive to work with a large variety of products, to achieve a high quality supply and a reliability and environmental standards, to take into account fast appearance of new products and increasing competitiveness of the companies, to apply information technologies. In other words today's market is like a highly dynamic environment and accordingly this supply chains should correspond highly dynamic systems. Different mathematical models for describing SC dynamic behavior were introduced in recent times. Part of them is used for optimization of operating SC and application of control theory methods for solving mentioned problems, for example [9],[6],[8].


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Many companies primarily take into account cost saving and compliance with deadlines when unforeseen events happen. For example these events could occur due to transfer and storage of products. Accordingly this they attempt to analyze ability of their system to react on time and to manage SC in this cases.

Modern technical capacity of receiving information about SC current condition enable to improve SC operating. Thus one of the main challenges in the SC management is SC execution problem in the case of external disturbances [3], [2].

In this paper the problem of construction of the multiprogrammed control for desired (conducive) programmed regimes is considered. We suggest to use positional optimization method and linear programming methods for solving this problem. To demonstrate proposed methods, we applied them to the model of one supply chain.

## 2. MODEL DESCRIPTION

Let us consider the multiprogrammed control problem of one SC. Following [1], we consider a mathematical model of generalized supply chain functioning.

It is assumed that supply chain consist of three elements: manufacturer (plant), a warehouse and the customer (Figure 1). Under the plant we mean one or more plants of the same manufacturer, which produced only one type of products manufactured with the necessary power. This product comes from the factory to the warehouse, and from there It delivered to the customer with the corresponding demand level. Consumer here can be a wholesale manufacturer, dealer companies and end consumers. It is assumed that the work of the supply chain changes dynamically.

In this paper, using this model [1], we'll find the optimal production rate that provides a predetermined level of demand, taking into account incoming information about inventory level in the warehouse. $I(t)$−the inventory level at

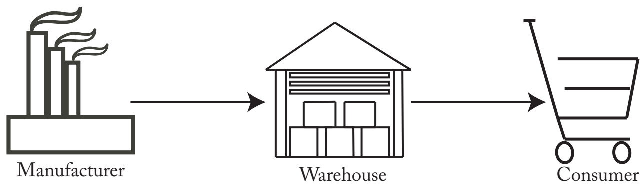


**Figure 1: Operational scheme of considered supply chain**

time t, $P_a(t)-$the actual production rate at time t, $D(t)-$the demand rate at time t, $u(t)-$the desired production rate at time t, $\theta(t)-$the deterioration rate, $\alpha-$the inverse of exponential delay time. All functions are assumed to be nonnegative, continuous and differentiable. Therefore first order differential equations of considered model are following:

$$\begin{cases} \dot{I}(t) = P_a(t) - \theta(t)I(t) - D(t) \\ \dot{P_a}(t) = \alpha(t)(u(t) - P_a(t)) \end{cases} \tag{1}$$

Let us introduce the following notation:

$$\frac{d}{dt}\begin{pmatrix} I \\ P_a \end{pmatrix} = \begin{pmatrix} -\theta & 1 \\ 0 & -\alpha \end{pmatrix}\begin{pmatrix} I \\ P_a \end{pmatrix} + \begin{pmatrix} 0 \\ \alpha \end{pmatrix}u(t) + \begin{pmatrix} -D(t) \\ 0 \end{pmatrix},$$

$$\begin{pmatrix} I \\ P_a \end{pmatrix} = x(t), \begin{pmatrix} -\theta & 1 \\ 0 & -\alpha \end{pmatrix} = A(t), \begin{pmatrix} 0 \\ \alpha \end{pmatrix} = b(t), \begin{pmatrix} -D \\ 0 \end{pmatrix} = f(t).$$

For brevity sake (1) could be rewrite in the form:

$$\dot{x}(t) = A(t)x(t) + b(t)u(t) + f(t), \tag{2}$$

where $A(t) - n \times n$ matrix; $b(t) - n$-dimensional vector function; $u(t)-$the value of control action; $f(t)-$piecewise continuous bounded disturbance.

The model parameters could be defined by identifying the model using the statistical information about demand, output volumes and the inventory level on the previous periods of time of SC work. For example, the products arrival rate from the warehouse to the consumer could be received from the statistical information about turnover of a warehouse and the estimates of demand for products for the previous period. The input parameters of the model are defined with a certain accuracy, so the results of numerical experiments for different values of the parameters could help the manufacturer to decide the value of the release.

## 3. PROBLEM STATEMENT

Let us consider the supply chain (Fig. 1), consisted of the manufacturer, consumer and warehouse. Assume that the desired programmed regimes (schedule of SC operation) $x_i(t)$, $i = \overline{1,n}$, for system (2) are provided with programmed controls $u_{pi}(t)$, $i = \overline{1,n}$. To realize $n$ different desired programmed regimes $x_i$ we can construct a multiprogrammed control [**?**]

$$u_{mp}(x(t), t) = \sum_{i=1}^{n} u_{p_i}(t) \prod_{j \neq i} \frac{(x(t) - x_j(t))^2}{(x_i(t) - x_j(t))^2}, \tag{3}$$

where scalar product is denoted as $(x_i(t)-x_j(t))^2$. The main property of this multiprogrammed control (3) is $u_{mp}(x_i(t), t) = u_{p_i}(t)$, $i = \overline{1,n}$.

Also suppose that the manufacturer has the opportunity to obtain information about the level of inventory in stock and compare it with the actual rate of production at fixed moments of time. At these moments, the manufacturer can change the production strategy (control the value of release) to provide the expected level of demand and maximize profits at the end of the planning period of time. Since the planned volume of supply may differ from current, the manufacturer has to regularly adjust the work of the supply chain. We assume that the control $u$ belongs to the class of piecewise constant functions, that is, the manufacturer can change the volume of release (control $u$) at certain times. Thus, the aim is to find an optimal control $u$ of the SC described by the system (2), providing the expected level of demand at the moment of time $t^*$, given by constraints: $Hx(t^*) = g, g \in R^m, rankH = m < n$ and maximizing profits, described by $c'x(t) \to max$. Furthermore, another our goal is to conduct numerical experiments for different values of the parameters.

## 4. METHODS

Let us design optimal control: $u(x(t), t) = u_{mp}(x(t), t) + v(t)$. At first for each programmed regime $x_i(t), i = \overline{1,n}$ vectors $y_i(t) = x(t) - x_i(t)$, $i = \overline{1,n}$ could be considered and a deviation system for $i = \overline{1,n}$ could be obtained. In view of multiprogrammed control (3) each of them is nonlinear system.

In this study we propose to use Positional optimization method [7] for solving the SC multiprogrammed control problem. This method is based on widely known mathematical methods of optimal control problems and linear programming (adaptive method) [5]. It enables to solve control problems of both linear and nonlinear systems and it was applied in a number of cases, for example [7], [10], [4]. Adaptive method algorithms offers variety of advantages, because it is finiteness, exact and relaxation, algorithm. Moreover one important fact that it focuses on the linear programming problems with banded structure of matrix, that are appear when there are a large number of segments $N$ of considered time interval $[t_*, t^*]$. Here it permits us to get a current value of positional solution and calculate $v(t)$ in control process.

The main idea of this method resides in sequential designing of optimal programmed control (OPC) on the time interval $T$. In this process current entry information about system state is taken into consideration at each period of time. Let $T = [t_*, t^*]$, $h = (t_* - t^*)/N$, $t_* < t^* < +\infty$ $T_u = \{t_*, t_* + h, \ldots, t^* - h, t^*\}$, function $v(t)$, $t \in T$ define as a discrete control, if $v(t) = v(t_* + kh)$, $t \in [t_* + kh,\ t_* + (k+1)h]$, $k = \overline{1, N-1}$. In this class of discrete controls we consider the auxiliary OPC problem with one of achieved deviation systems at the moment of time $\tau = t_* + kh$ on the time interval $T_\tau = \{\tau, \tau + h, \ldots, t^* - h, t^*\} \in T_u$.

The solution of the auxiliary OPC problem can be divided into two steps: the reduction of an optimal control problem for interval linear programming problem and solving interval linear programming problem by adaptive method.

## 5. NUMERICAL EXAMPLES

Above described procedure was realized in MATLAB. It was verified on the systems with not large dimensions. For the SC with input parameters from Table 1 we calculate optimal positional control $v(t)$. It is demonstrated on the Figure 2.

**Table 1: Input Parameters**

| Input Parameters | | | | | | |
|---|---|---|---|---|---|---|
| $A$ | | $b$ | $f$ | $c$ | $t_*$ | $t^*$ |
| -0.05 | 1 | 0 | $-2\cos t_i$ | -2 | 0 | $\pi$ |
| 0 | -5 | 5 | 0 | 2 | | |

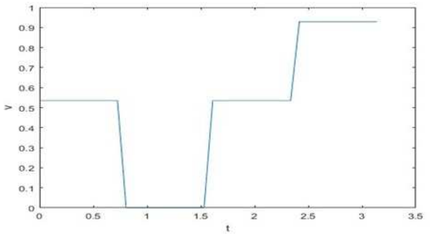


**Figure 2: Values of optimal positional control $v(t)$ in numerical example**

# 6. CONCLUSION

In this paper the problem of designing multiprogrammed control of one SC is considered. Here we propose the approach to design optimal control of one SC with the availability several early picked desired regimes and in case it is subject to external actions. Results are verified with the help of this procedure realization on numerical examples. In prospect we are going to apply this solution strategy to the model of real logistics or manufacturer company and to test it on real data. Additionally, we are eager to compare results of our modeling with the results obtained using other methods of SC control on real data.

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