

# OD-Matrix Estimation for Urban Traffic Area Control

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## ABSTRACT

Congestion, accidents, greenhouse gas emission and others seem to become unsolvable challenges for all levels of management in modern worldwide large cities. The increasing dynamics of motorization requires development of innovative methodological tools and technical devices to cope with problems appeared on the road networks. Primarily, control system for urban traffic area has to be created to support decision makers via operating with big transportation data. The input for a such system is a volume of travel demand between origins and destinations – OD-matrix. The present work is devoted to the problem of OD-matrix estimation. The original technique of plate scanning sensors location is offered. The developed method has been tested on the experimental parameters of the Saint-Petersburg road network.

## 1. INTRODUCTION

The problems of estimation and reconstruction of trip matrices are difficult and actual problems in transportation researches. In general, trip matrices estimation and reconstruction are different problems and their solutions can be unequal [1]. One of the first mathematical models of a trip matrix estimation developed in the end of XX century was formulated as a bi-level program [2]. Despite numerous publications, this problem is still pressing scientific issue that require further research [8]. Among the recent results in this area the paper [3] should be mentioned, since in this article authors consider the problem of trip matrix and path flow reconstruction and estimation by combining data from plate scanning and link flow observation. A detailed comparative analysis of the three methods of trip matrix estimation (a method of linear programming, Bayesian approach, the

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method of time-varying network tomography) was made in [4]. The actual size of transportation networks are very large and it is reasonable to formulate the problem of minimizing the number of sensors [5, 6]. In [7] a new model based on Total Demand Scale is developed.

The important practical implications on an efficient estimation of traffic flows by using plate scanning sensors was made by [3]. However, the problem of the optimal sensors location in terms of maximum observation of traffic flows has been never considered. This paper deals with a such formulation and, simultaneously, proposes a model of plate scanning sensors location, that ensures the effective application of the Castillo method [3].

## 2. THE OPTIMAL PLATE SCANNING SENSORS LOCATION

We assume that the plate scanning sensor could identify the plate of a vehicle on the lane where the sensor has been installed. Consider the transportation network presented by digraph  $G = (N, A)$ ,  $N$  — set of vertices,  $A$  — set of arcs. We introduce the following notation:  $W$  — set of OD-pairs,  $w \in W$ ;  $K^w$  — set of routes between OD-pair  $w$ ,  $w \in W$ ;  $q_a$  and  $c_a$  — number of sensors and number of lanes on the arc  $a \in A$  corresponding;  $q = (\dots, q_a, \dots)$ ;  $A_k$  — set of arcs belonging to the route  $k \in K^w$ ,  $A_k \subset A$ ;  $\bar{f}_k$  — *a priori* traffic flow through route  $k \in K^w$ . The probability of identification of a vehicle on the arc  $a \in A$  with  $c_a$  lanes is  $\frac{q_a}{c_a}$ ,  $\forall a \in A$ .

We assume that the random events of plate identification on a sequence of arcs are independent. The goal could be formulated as follows:

$$\max_q z(q) = \max_q \sum_{r \in R} \sum_{s \in S} \sum_{k \in K^{rs}} \left( \prod_{a \in A_k} \frac{q_a}{c_a} \right) \bar{f}_k, \quad (1)$$

where  $R$  is for origins and  $S$  — for destinations.

Thus we maximize the probability of fixing traffic flows throughout the route. At the same time, the condition that at least one arc in each path is scanned, could be given by the following inequality:

$$\sum_{a \in A_k} q_a \geq 1 \quad \forall k \in K^{rs}. \quad (2)$$

The existence of at least one scanned arc, belonging to the route  $k_1$  and not belonging to any other route  $k_2$ , is guar-

anted by the following constraint:

$$\sum_{a \in A} q_a \delta_a^{k_1 k_2} \geq 1 \quad \forall k_1, k_2 \in K^{rs} : k_1 \neq k_2, \quad (3)$$

$$\delta_a^{k_1 k_2} = \begin{cases} 1, & \text{if } a \in k_1, a \notin k_2, \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

Consider the budget constraint, where  $Q$  — the number of available sensors:

$$\sum_{a \in A} q_a \leq Q. \quad (5)$$

The number of sensors on the arc should not exceed the number of lanes:

$$0 \leq q_a \leq c_a, \quad \forall a \in A.$$

So, we formulated the integer program on a limited set of solutions. The solution of this problem exists when the set of possible solutions is not empty.

### 3. COMPUTATIONAL EXPERIMENT

Consider a multi-lane traffic network of Saint-Petersburg city center. We define nine areas of origin-destination (fig. 1), nine routes between pairs of origins and destinations and, hence, 21 arcs (fig. 2).

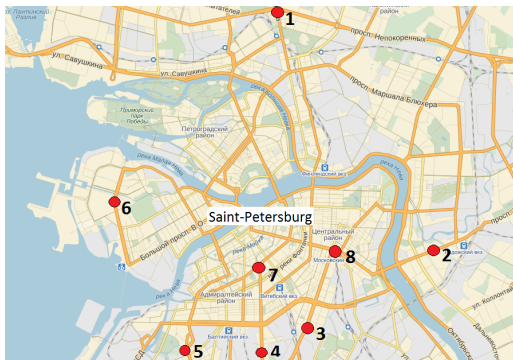


Figure 1: Origins-destinations

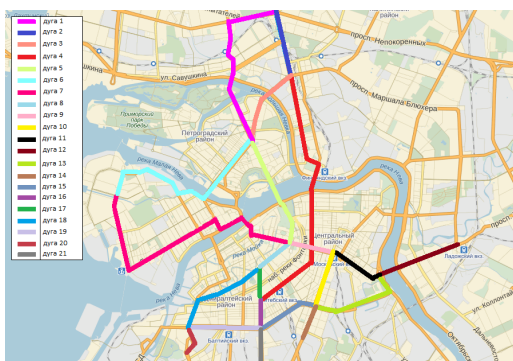


Figure 2: Links

We define the number of lanes  $c_a$  and *a priori* traffic flows  $\bar{f}_k$  on the available routes. Then we calculate optimal plate scanning sensors location for the different budget constraints

varying budget values from 0 to 50. Since there is the optimization problem with nonlinear functional and linear constraints, we employ *fmincon*-function in a software environment MatLab. The dependence between the observed flows value and the number of sensors located on the network are presented on the fig. 3.

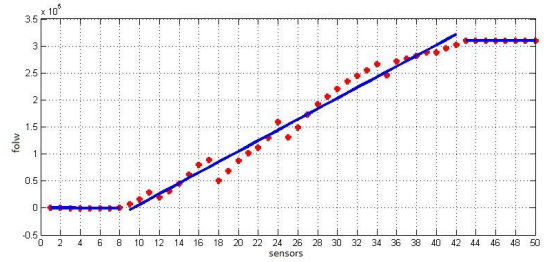


Figure 3: Total flow

Consider the different combinations of constraints — separately (5), then (5) and (2), and then all constraints (2) — (5). The dependence between the value of the total observed flow and the number of sensors on a network for different combinations of constraints does not change. Therefore, this analysis was performed for each of considered routes. If we analyze the dependence between the number of sensors on the network and the value of observed flows then we recognize that routes with significant flows and small number of arcs are observed primarily. By increasing the amount of constraints large flows on routes with lots of arcs and lanes begin to occur later (fig. 4).

Number of sensors on the network	The proportion of the observed flow f(1), one constraint.	The proportion of the observed flow f(1), two constraints.	The proportion of the observed flow f(1), all constraints.
5	0	0	0
10	0	0	0
15	1,00	0	0
20	1,00	1,00	0
25	1,00	1,00	1,00
30	1,00	1,00	1,00
35	1,00	1,00	1,00
40	1,00	1,00	1,00
45	1,00	1,00	1,00
50	1,00	1,00	1,00

Figure 4: Combinations of restrictions

### Conclusion

The article is devoted to the problem of optimal location of traffic plate scanning sensors on the network. A brief review of the literature is carried out and the method of Castillo is recognized. To increase the efficiency of the Castillo method, an original model of optimal location of plate scanning sensors on the network of general topology is developed. The developed model allows to maximize probability of identification of the most significant traffic flows throughout the corresponding routes. We obtained the following theoretical and practical results:

- a deterministic model of optimal plate scanning location on the network of general topology;
- an algorithm for solving deterministic problem of the sensors location;

- implementation of developed algorithm on the transportation network of St. Petersburg.

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