A Program Complex for Learning of Optimal Control Problems

Andrey Klyuenkov
Saint-Peterburg State University
7-9, Universitetskaya nab.
Saint-Petersburg, 199034, Russia
andr_esk@mail.ru

ABSTRACT

In the present paper the task is to develop a learning program complex on optimal control problems. For solving optimal control problems the adaptive algorithm of R. Gabasov is used. The structure of a program complex for continuous models is proposed. The solution is divided into two stages: reducing the optimal control problem to an interval linear programming problem (ILPP), and applying the adaptive method to the ILPP. The developed complex is implemented in MATLAB environment. Stabilization of three-mass oscillatory system with minimal fuel consumption problem is used for testing. The optimal control is computed, and results are presented graphically.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search — *control theory*

General Terms

Theory

Keywords

Optimal control problem, interval linear programming problem, learning

1. INTRODUCTION

In many control problems we have to find the optimal control. Often this problem is quite difficult, however for controllable systems the solution can be found with the adaptive method by R. Gabbasov [1, 2, 3]. Teaching students this method and its program implementation gives them a way to solve optimal control problem and opens a new viewpoint towards the problems. Furthermore this contains unconventional approach to the linear programming problem.

The task was to develop a learning complex on optimal control problems.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.
IWAIT '15, Oct. 8 – 10, 2015, Aizu-Wakamatsu, Japan.
Copyright 2015 University of Aizu Press.

2. STRUCTURE OF THE COMPLEX

Problem statement. The adaptive method can be applied to differential as well as to discrete systems. Consider a system of ordinary differential equations with a control parameter and given initial conditions

$$\dot{x} = A(t)x + b(t)u, \tag{1}$$

$$x(t_*) = x_0, \tag{2}$$

$$t \in [t_*, t^*] = T, \quad t_* < t^* < +\infty,$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^1$, $A(t)$ is a $(n \times n)$-matrix, $b(t)$ is a vector function whose elements are piecewise continuous functions for $t \in T$.

The function $u(t)$, $t \in T$ denotes a *discrete control* (with the quantization step h), if

$$u(t) = u(t_* + kh) = u_{kh}, \quad t \in [t_* + kh, t_* + (k+1)h[9 \tag{3}$$

for every $k = \overline{0, N-1}$. Here $h = (t^* - t_*)/N$ is the quantization step and N is a natural number. Besides, $|u(t)| \leqslant L$ for $t \in T$, $L > 0$. Denote the set of decomposition points as $T_u = \{t_*, t_* + h, \ldots, t^* - h\}$.

Let's formulate a terminal control problem: To find the solution of the initial value problem (IVP) (1), (2) with piecewise constant control, that maximizes the objective function

$$c^T x(t^*) \to \max \tag{4}$$

provided that

$$Hx(t^*) = g, \tag{5}$$

where $g \in \mathbb{R}^m$ and $\operatorname{rank} H = m < n$.

The problem is solved with the adaptive Gabasov method. The method is applied in two stages. *The first stage* is reducing of the optimal control problem to an *interval linear program problem* (ILPP). *The second stage* is solving the obtained ILPP with the adaptive method.

The first stage can be presented as following. The general solution of the IVP (1),(2) at time $t = t^*$ is

$$x(t^*, t_*, x_0) = Y(t^*) \left[Y^{-1}(t_*)x_0 + \int_{t_*}^{t^*} Y^{-1}(\tau)b(\tau)u(\tau)d\tau \right], \tag{6}$$

where $Y(t)$ is the fundamental matrix of the corresponding homogeneous system. The substitution of (6) to (4) gives an equivalent representation

$$\int_{t_*}^{t^*} c^T Y(t^*) Y^{-1}(\tau) b(\tau) u(\tau) d\tau \to \max. \tag{7}$$

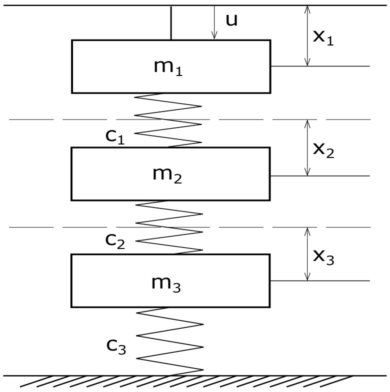

Figure 1: Oscillatory system

Consider the dual system with initial conditions

$$\dot{z} = -A^T(t)z, \tag{8}$$
$$z(t^*) = c, \tag{9}$$

where c and t^* comes from the conditions (4), (5). Its solution can be written as $z_c(t) = Z(t)Z^{-1}(t^*)c$, where $Z(t)$ is the fundamental matrix of the system (8). The main property of the fundamental matrix is $Z^T(t) = Y^{-1}(t)$. With use of this property it's easy to find that

$$z_c^T(t) = c^T \left(Z^{-1}\right)^T (t^*)Z^{-1}(t) = c^T Y(t^*)Y^{-1}(t). \tag{10}$$

Considering (10) the condition (7) becomes

$$\int_{t_*}^{t^*} z_c^T(\tau)b(\tau)u(\tau)d\tau \to \max. \tag{11}$$

Now consider the terminal condition (5). Using (6) it can be represented as

$$\int_{t_*}^{t^*} HY(t^*)Y^{-1}(\tau)b(\tau)u(\tau)d\tau = g_0, \tag{12}$$

where $g_0 = g - HY(t^*)Y^{-1}(t_*)x_0$ is a vector of size m.

Taking into account that we search for control in the form (3), we can rewrite (11) and (6) as

$$\sum_{t\in T_u} c_k u(t) \to \max, \tag{13}$$
$$\sum_{t\in T_u} d_k u(t) = g_0, \tag{14}$$

respectively, where

$$d_k = \int_t^{t+h} HY(t^*)Y^{-1}(\tau)b(\tau)d\tau, \quad c_k = \int_t^{t+h} z_c^T(\tau)b(\tau)d\tau,$$

for $t = t_* + kh$, $k = \overline{0, N-1}$.

Let's denote

$$U = (u_0, u_1, \dots, u_{kh}, \dots, u_{(N-1)h})^T,$$

$$C = (c_0, c_1, \dots, c_{N-1}), \quad D = (d_0, d_1, \dots, d_{N-1}).$$

Finally the problem (13), (14) can be represented in the form

$$\begin{cases} C^T U \to \max, \\ DU \leq g_0, \\ |u_k| \leq L, \quad k = \overline{1, N}. \end{cases} \tag{15}$$

We give a short outline of the second stage following [4]. It's important since the package of applied programs, that implement the construction of optimal control, was written on its base.

Often it is impossible to get exact results of measuring on practice, in which case it's unnecessary to find the exact solution. The adaptive method can find approximate (suboptimal) solutions [1, 4].

Let's show its general scheme by the example of a general ILPP

$$\varphi(x) = c^T x \to \max, \quad b_* \leqslant Ax \leqslant b^*, \quad d_* \leqslant x \leqslant d^*. \tag{16}$$

We need to find an ε-optimal solution x^ε, $\varepsilon \geqslant 0$, using an a priori known information about the solution of the problem(16). To find the solution we construct a sequence of approximations $\{x^i\}$, $i = 1, 2, \dots$, which are plans of the problem (16). A basic instrument of the adaptive method are *supports*. Thereby a sequence of approximations and a sequence of supports $\{K_{op}^i\}$, $i = 1, 2, \dots$ are constructed. At every iteration of the method a new pair is constructed

$$\{x^k, K_{op}^k\} \to \{x^{k+1}, K_{op}^{k+1}\}.$$

The method consists of two phases. At the first phase on base of the known information about the problem (16) solution a first plan $\{x^1, K_{op}^1\}$ is constructed. The construction of following approximations is the second phase of the adaptive method.

In the adaptive method at every step it's possible to find the target value deviation, named suboptimality estimation and denoted $\beta(x^k, K_{op}^k)$. The method is based on the principle of suboptimality reduction:

$$\beta(x^{k+1}, K_{op}^{k+1}) \leqslant \beta(x^k, K_{op}^k).$$

The estimation of suboptimality allows the representation

$$\beta(x^k, K_{op}^k) = \mu(x^k) + \mu(K_{op}^k),$$

so two separate procedures can be done:

1) plan update $x^k \to x^{k+1}$, and

2) support update $K_{op}^k \to K_{op}^{k+1}$.

It is proven [4] that the adaptive method of finite number of steps converge to optimal solution.

The method was implemented in MATLAB environment and is suitable for solving various optimal control problems.

Note that the developed algorithm has some drawbacks. This method can converge slow enough if dimension of the system will be high. Also in the last iteration the rate of convergence reduced, especially for small values of ε.

3. EXAMPLE

As an example we consider the problem of fixed time stabilization of a three-mass oscillatory system with minimal fuel consumption (Fig. 1).

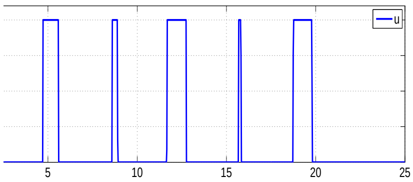

Figure 2: Optimal control

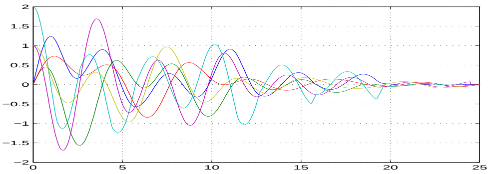

Figure 3: State variables. Transient

A mathematical model of the problem is

$$\begin{cases} \dot{x}_1 = x_4, \\ \dot{x}_2 = x_5, \\ \dot{x}_3 = x_6, \\ \dot{x}_4 = -2.33x_1 - x_2 + 0.33x_3 + u, \\ \dot{x}_5 = -0.88x_1 - 2.06x_2 - 0.88x_3, \\ \dot{x}_6 = 0.25x_1 - 0.75x_2 - 1.75x_3, \end{cases}$$

$x_1(0) = x_2(0) = x_3(0) = 0,\ x_4(0) = 2,\ x_5(0) = 1,\ x_6(0) = 1,$ $x_1(25) = x_2(25) = x_3(25) = x_4(25) = x_5(25) = x_6(25) = 0,$

$$\int_0^{25} u(t)dt \to \min, \quad 0 \leqslant u(t) \leqslant 1, \quad t \in [0, 25[,$$

where $x_1 = x_1(t)$, $x_2 = x_2(t)$, $x_3 = x_3(t)$ are deviations from the equilibrium position of the first, second and third masses, respectively, and $u = u(t)$ is the control parameter.

This optimal control problem with the quantization period $h = 0.025$ is equivalent to a linear programming problem of dimension 6×1000. The adaptive method was used to construct the optimal control. The problem is solved in 141 iterations (Figures 2, 3).

4. CONCLUSION

A learning software system on optimal control problems is described. The stages of the complex are presented, the software package implementing the method is written. It provides the tools for visualiazation of the the adaptive Gabbasov method work, and for optimal control construction for arbitrary mathematical models.

The software package is used and developed at a special seminar of the Department of Mathematical Modelling of Economical Systems, Faculty of Applied Mathematics and Control Processes, Saint-Petersburg State University.

In the next stage to expand the methodological basis of the program complex we are planning to use the results and examples of works [5–13].

5. REFERENCES

[1] R. Gabasov and F. M. Kirillova. *Constructive Optimization Methods. P. 2. Control Problems*. Minsk, USSR, 1977. (in Russian)

[2] N. V. Balashevich, R. Gabasov, and F. M. Kirillova. Numerical methods of programmed and positional optimization for linear systems. *Journal of Computational Mathematics and Mathematical Physics*, 40(6):838–859, 2000. (in Russian)

[3] R. Gabasov, F. M. Kirillova, and E. A. Rugitskaya. Dampening and stabilization of equilibrium with great initial disturbance. *Intelligence of Academy of Science. Theory and Systems of Control*, No 1: 29–38, 2001. (in Russian)

[4] V. V. Al'sevich, R. Gabasov and V.S. Glushenkov. *Optimization of Linear Economic Models. Static Problems.* Belarus State University Press, Minsk, 2000. (in Russian)

[5] I. Solovyeva. Positional optimization in a certain problem of multiprogrammed control. In *Proceedings of the 11-th International Conference on Humans and Computers*, pages 359–393, University of Aizu Press, Japan, November 2008.

[6] N. V. Smirnov and I. V. Solovyeva. Application of a closed-loop optimization method for multiprogram stabilization of bilinear systems. *Vestnik of St. Petersburg University. Ser. 10: Applied Mathematics. Informatics. Control Processes*, (3):251–259, September 2009 (in Russian).

[7] A. Zgonnikov. The optimum feedback synthesis for a certain nonlinear mechanical system. In *Proceedings of the 12-th International Conference on Humans and Computers*, pages 235–239, University of Aizu Press, Japan, December 2009.

[8] M. Smirnov. Suppression of Bounded Exogenous Disturbances Act on a Sea-going Ship. In *Proceedings of the 13-th International Conference on Humans and Computers*, pages 114-116, University of Aizu Press, Japan, December 2010.

[9] M. Fedorova. Computer Modeling of the Astatic Stabilization System of Sea-going Ship Course. In *Proceedings of the 13-th International Conference on Humans and Computers*, pages 117-120, University of Aizu Press, Japan, December 2010.

[10] N. Smirnov. Multiprogram Control for Dynamic Systems: a Point of View. In *Proceedings of the Joint International Conference on Human-Centered Computer Environments (Aizu-Wakamatsu, Japan, Duesseldorf, Germany, HCCE 2012)*, pages 106–113, ACM, New York, NY, March 2012. DOI=10.1145/2160749.2160773.

[11] N. V. Smirnov, T. E. Smirnova, Multiprogram digital control for bilinear object. *2014 International Conference on Computer Technologies in Physical and Engineering Applications (ICCTPEA). Editor: E.I. Veremey*, pages 168–169, St. Petersburg State University, Russia, June–July 2014.

[12] N. V. Smirnov, T. E. Smirnova and Ya. A. Shakhov. Stabilization of a given set of equilibrium states of nonlinear systems. *Journal of Computer and Systems Sciences International*, 51(2):169–175, April 2012.

[13] N. V. Smirnov and T. Ye. Smirnova. The synthesis of multi-programme controls in bilinear systems. *Journal of Applied Mathematics and Mechanics*, 64(6):891–894, December 2000.