Multicriteria Regulation of Investments in the Economy of the Russian Federation

Alexander Popkov
Saint-Peterburg State University
199034, Universitetskaya nab., 7-9
Saint-Petersburg, Russia
alexandr.popkoff@gmail.com

ABSTRACT

In the present paper the problem of optimal distribution of investments to the sectors of the economics is considered. A dynamic input-output model for describing the development of multi-commodity economics is used. On the base of the available input-output tables (Federal State Statistics Service, Rosstat) a system of differential equations with a control vector in the right-hand side was constructed. There were two goals of regulation: to maximize the GDP and to minimize the value of investments. To solve the multicriteria task the regulation problem was reduced to a linear programming problem. Four different choice principles were considered. For each of them the optimal plan of investments was found.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search — *control theory*

General Terms

Theory

Keywords

Optimal control, multicriteria problems, linear programming, input-output model, economy

1. INTRODUCTION

The theoretical foundations of the input-output model were provided by works of Nobel laureates in Economics W. W. Leontief and L. V. Kantorovich [1, 2]. Currently, the input-output model is one of the internationally acknowledged scientific instrument for analysis of regional economic systems and macroeconomic trends in these systems. The International Input-Output Association [3], which brings together scientists concerned with the theory and practice of application of input-output models, has existed and has been actively functioning for 25 years. In Russian Federation, the statistics on changes in the coefficients of input-output tables are published annually by Rosstat [4].

In this work the dynamic input-output model [5] is used for describing development of a multicommodity economy. The problem of distribution of investments in sectors of economy of the Russian Federation is considered. Data for this model for 2004 — 2006 years are obtained from the official site of Rosstat [4]. The proposed model contains a control vector — quarterly investments in sectors of the economy. The investment process has two objectives: to maximize the GDP and to minimize value of investments. To solve this multicriteria task we use the approach of the multi-objective optimization theory [6].

2. PROBLEM STATEMENT

The dynamic of the main indicators of Russian economy in 2006 can be described by the following linear system of differential equations with control vector [5, 7]:

$$\begin{gathered} \dot{I} = D_{2006} I + Q_{2006} u, \\ 0 \leqslant u_i \leqslant L_i, \quad i = \overline{1, n}, \\ I(0) = I_{2006}, \end{gathered} \tag{1}$$

where the first $n-1$ components of I are the volumes of outputs by sectors in terms of money, I_n — GDP. In this paper there are three aggregated sectors: industry, life support and infrastructure. Data from other sectors are included in the process of reduction, u — n-dimensional vector of investment in the industry, D_{2006}, Q_{2006} — $(n \times n)$ – matrices, obtained using the input-output tables for 2006 [4], I_{2006} — output and GDP vector for 2006:

$$I(2006) = \begin{pmatrix} 13.784 \cdot 10^{12} \\ 5.439 \cdot 10^{12} \\ 14.500 \cdot 10^{12} \\ 21.619 \cdot 10^{12} \end{pmatrix}.$$

Consider the regulation process for one year: $t \in [0, 1]$. Let investment is flowing in the economy on a quarterly basis. Therefore the control parameters can be represented as

$$u_i(t) = u_i^j, \quad (j-1)/4 < t \leqslant j/4, \quad \forall i = \overline{1, 4}. \tag{2}$$

Now we introduce two target functionals:

- maximization of GDP by the end of the year: $I_4(1) \to \max$,
- minimization of expenses: $\int_0^1 (\sum_{i=1}^4 u_i) dt \to \min$.

Let us assume these objectives are equivalent.

3. MULTICRITERIA OPTIMIZATION

By writing the solution of differential equations (1) in the form of Cauchy, at $t = 1$:

$$I(1) = e^D \left(I(0) + \int_0^1 e^{-Dt} Qudt \right),$$

and by replacing the control u using representation (2), the problem can be reduced to a linear programming problem with two objective functions:

$$\begin{aligned} f_1(x) &= c_1^T x \to \max_u, \\ f_2(x) &= c_2^T x \to \min_u, \\ 0 \leqslant x_i &\leqslant L_i, \quad i = \overline{1, 4n}, \end{aligned} \tag{3}$$

where $x_{4(i-1)+j} = u_i^j$.

At first we will find the set of admissible plans as the locus that satisfying to restrictions of a task and designate this set by symbol X^*. Obviously that in this task this set is represented by the 16-dimensional parallelepiped.

Next we will carry out the procedure of natural normalization for the functions considered:

$$g_i(x) = \frac{f_i(x) - \min_{x \in X^*} f_i(x)}{\max_{x \in X^*} f_i(x) - \min_{x \in X^*} f_i(x)}, \quad i = 1, 2.$$

Now the values of both functions are measured in dimensionless sizes, their minima are equal to 0, maxima — to 1. Because of the first functional is need to tend to the maximum, and the second to a minimum, we consider auxiliary functions $h_1(x)$ and $h_2(x)$:

$$h_1(x) = g_1(x), \quad h_2(x) = 1 - g_2(x).$$

Now we will enter the set

$$Y^* = \{Y = (y_1, y_2) \mid y_1 = h_1(x),\ y_2 = h_2(x),\ x \in X^*\}.$$

Let's say that Y' is more preferable than a vector Y'' ($Y' \succ_Y Y''$), if $y_1' \geqslant y_1''$ and $y_2' \geqslant y_2''$, and one of these inequalities is strong. We will call Y' non-dominated on the set Y^*, if in this set there is no such vector Y, that $Y' \succ_Y Y$. We denote the set of non-dominated vectors (the set of Pareto-optimal solutions) as $P(Y^*)$.

Consider the vectors $Y', Y'' \in P(Y^*)$. Let $|y_1' - y_1''| = w_1$, $|y_2' - y_2''| = w_2$. We introduce coefficients of the relative importance:

$$\theta_1 = \frac{w_1}{w_1 + w_2}, \quad \theta_2 = \frac{w_2}{w_1 + w_2}.$$

Further we will enter a set of relative important vectors:

$$V(Y^*) = \{Y' \in P(Y^*) |\ \theta_1 \geqslant \theta_1^*,\ \theta_2 \geqslant \theta_2^*,\ \forall Y'' \in P(Y^*)\},$$

$\theta_1^*,\ \theta_2^*$ are advance set coefficients.

Obviously that $V(Y^*) \subseteq P(Y^*) \subseteq Y^*$. We will look for the solution of a problem on a set $V(Y^*)$.

Algorithm has some drawbacks. For solution is needed to find set X^*. It's very difficult or impossible even for linear restrictions of the form $Ax = b$. Therefore used only box restrictions $0 \leqslant x_i \leqslant L_i$.

4. PRINCIPLE OF A CHOICE

To find the optimal vector $Y^0 = (y_1^0, y_2^0)$ we will use the following principles of choice:

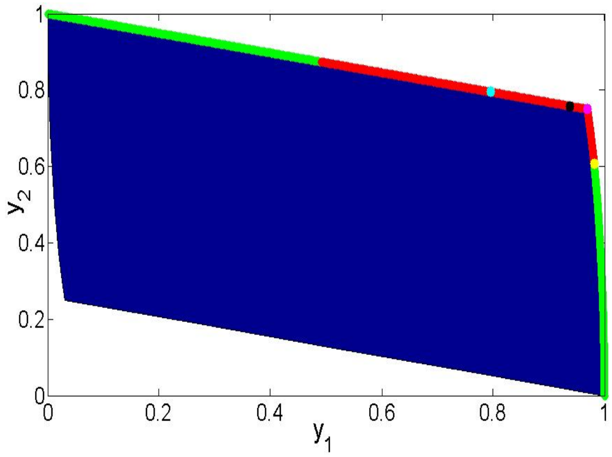

Figure 1: The space of values of target functions

1. the principle of dominant result : $\max(y_1^0, y_2^0) = \max_{V(Y^*)} \max(y_1, y_2)$
2. the principle of aggregated efficiency: $y_1^0 + y_2^0 = \max_{V(Y^*)}(y_1 + y_2)$
3. the principle of guaranteed results: $\min(y_1^0, y_2^0) = \max_{V(Y^*)} \min(y_1, y_2)$
4. the principle of least deviation: $||1 - Y^0|| = \min ||1 - Y||,\ \forall Y \in V(Y^*)$.

5. NUMERICAL REALIZATION

To solve this problem, the authors implemented a program in MATLAB covering the following methodology requirements:

1. Reduction of input-output tables from the 16×16 to 4×4.
2. Construction of a system of differential equations (calculation of matrix D, Q and vector I).
3. Reduction of regulation problem to a linear programming problem.
4. Alignment of target functions.
5. Construction sets $Y^*, P(Y^*)$ and $V(Y^*)$.
6. Finding the optimal vector Y^0 for each principle of choice.
7. The calculation of the optimal control u^0 and optimized functional values for each vector Y^0.

6. RESULTS

The following parameters values were taken for modeling:

$$L_i = 10^{12}, \quad \theta_1^* = 0.03, \quad \theta_2 = 0.2045.$$

Fig. 1 represents a target functions space. Blue color denotes the set Y^*, green color is used for $P(Y^*)$, red — for $V(Y^*)$. The optimal vector Y^0 for the 1st principle of choice is illustrated by yellow dot, for the 2nd — by purple dot, for the 3rd — by blue dot, for the 4th — by black dot.

	1st qr.	2nd qr.	3rd qr.	4th qr.
u_1^0	0	0	0	0
u_2^0	$2.5 \cdot 10^{11}$	0	0	0
u_3^0	$2.5 \cdot 10^{11}$	$0.725 \cdot 10^{11}$	0	0
u_4^0	$2.5 \cdot 10^{11}$	$2.5 \cdot 10^{11}$	$2.5 \cdot 10^{11}$	$2.5 \cdot 10^{11}$

Table 1: Optimal control for the principle of dominant results

The following values of the control vector and optimized functionals were obtained for optimal vectors Y^0:

$$I_4^0(1) = 23.336 \text{ trillion roubles}$$

$$\int_0^1 \left(\sum_{i=1}^{4} u_i^0 \right) dt = 1.57 \text{ trillion roubles}$$

	1st qr.	2nd qr.	3rd qr.	4th qr.
u_1^0	0	0	0	0
u_2^0	0	0	0	0
u_3^0	0	0	0	0
u_4^0	$2.5 \cdot 10^{11}$	$2.5 \cdot 10^{11}$	$2.5 \cdot 10^{11}$	$2.5 \cdot 10^{11}$

Table 2: Optimal control for the principle of aggregated efficiency

$$I_4^0(1) = 23.332 \text{ trillion roubles}$$

$$\int_0^1 \left(\sum_{i=1}^{4} u_i^0 \right) dt = 1 \text{ trillion roubles}$$

	1st qr.	2nd qr.	3rd qr.	4th qr.
u_1^0	0	0	0	0
u_2^0	0	0	0	0
u_3^0	0	0	0	0
u_4^0	$2.5 \cdot 10^{11}$	$2.5 \cdot 10^{11}$	$2.5 \cdot 10^{11}$	$0.7 \cdot 10^{11}$

Table 3: Optimal control for the principle of guaranteed result

$$I_4^0(1) = 23.268 \text{ trillion roubles}$$

$$\int_0^1 \left(\sum_{i=1}^{4} u_i^0 \right) dt = 0.82 \text{ trillion roubles}$$

$$I_4^0(1) = 23.320 \text{ trillion roubles}$$

$$\int_0^1 \left(\sum_{i=1}^{4} u_i^0 \right) dt = 0.9675 \text{ trillion roubles}$$

7. CONCLUSIONS

In this work two goals of regulation of the economy trends are considered. According to the available input-output tables the system of differential equations with a control parameter vector is constructed. Within the framework of the mathematical model the problem of optimal distribution of investments in the sectors of the economy was solved. The optimal regulation problem was reduced to the linear programming problem, because for problems of this class the approach of multi-criteria optimization is well developed. Four principles of a choice are used. For each of them the optimal plan of investments is created. The final result depends strongly on the selection of choice principle. It is impossible to select the best principle from the set of all kinds of principles. This choice depends on the character of the problem and objectives.

	1st qr.	2nd qr.	3rd qr.	4th qr.
u_1^0	0	0	0	0
u_2^0	0	0	0	0
u_3^0	0	0	0	0
u_4^0	$2.5 \cdot 10^{11}$	$2.5 \cdot 10^{11}$	$2.5 \cdot 10^{11}$	$2.175 \cdot 10^{11}$

Table 4: Optimal control of the principle of least deviation

8. REFERENCES

[1] W. W. Leontief. *Essays in Economics. Theory, research, facts and policy.* Politizdat, Moscow, 1990. (in Russian)

[2] L. V. Kantorovich. *Mathematical methods of organizing and planning production.* Publ. House of Lenigrad State University, Leningrad, 1939. (in Russian)

[3] International Input-Output Association (IIOA) [Electronic resource]. *The official website of the IIOA.* URL: http://www.iioa.org/ (date of reference: 17.05.2015).

[4] Federal State Statistics Service, Rosstat [Electronic resource]. *The official website of the Rosstat.* URL: http://www.gks.ru/free_doc/new_site/vvp/tab-zatr-vip.htm (date of reference: 17.05.2015).

[5] V. P. Peresada, N. V. Smirnov and T. E. Smirnova. Development control of a multicommodity economy based on the dynamical input-output model. *Vestnik of St. Petersburg University. Ser. 10: Applied Mathematics. Informatics. Control Processes*, (4):119–132, December 2014.

[6] V. D. Nogin. *Making decisions in many criteria.* Fizmatlit, Moscow, 2002. (in Russian)

[7] A. S. Popkov. Identification of dynamic input-output model for the Russian economy and the optimal investment distribution. *Control Processes and Stability*, 2(1):696–701, April 2015. (in Russian)