Kernel Method
カーネル法

Yan PEI/裴岩 /ペイ　イエン

Contents

- 1 Introduction/序論
- 2 Dual Presentation
- 3 Principal Component Analysis /主成分分析
- 4 Kernel Based Principal Component Analysis/カーネル法主成分分析
- 5 Linear Discriminant Analysis /線形判別分析
- 6 Generalized Discriminant Analysis / 判別分析
- 7 Hard-margin SVM / Hard-marginサポートベクターマシン
- 8 Soft-margin SVM / Soft-marginサポートベクターマシン
- 9 Linear Regression /線型回帰
- 10 SVR / サポートベクター回帰
- 11 Function Space, Hilbert Space /関数空間, ヒルベルト空間

Contents

- 1 Introduction
- 2 Dual Presentation
- 3 Principal Component Analysis
- 4 Kernel Based Principal Component Analysis
- 5 Linear Discriminant Analysis
- 6 Generalized Discriminant Analysis
- 7 Hard-margin SVM
- 8 Soft-margin SVM
- 9 Linear Regression
- 10 SVR
- 11 Hilbert Space

Machine Learning/機械学習

- Supervised Learning/教師あり学習
  - 7,8,9,10
- Unsupervised Learning/教師なし学習
  - 3,4,5,6
- Reinforcement Learning /強化学習(きょうかがくしゅう)
- Learning Theory
  - 11

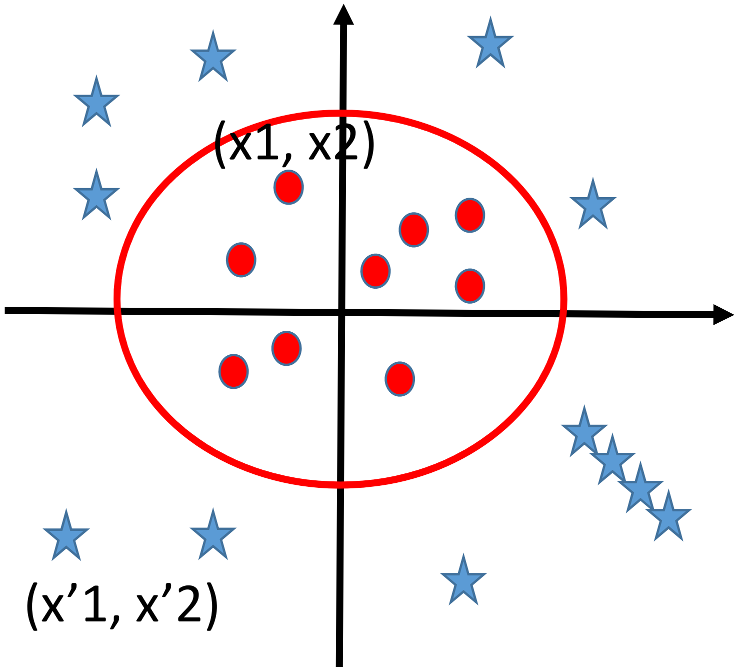
Introduction

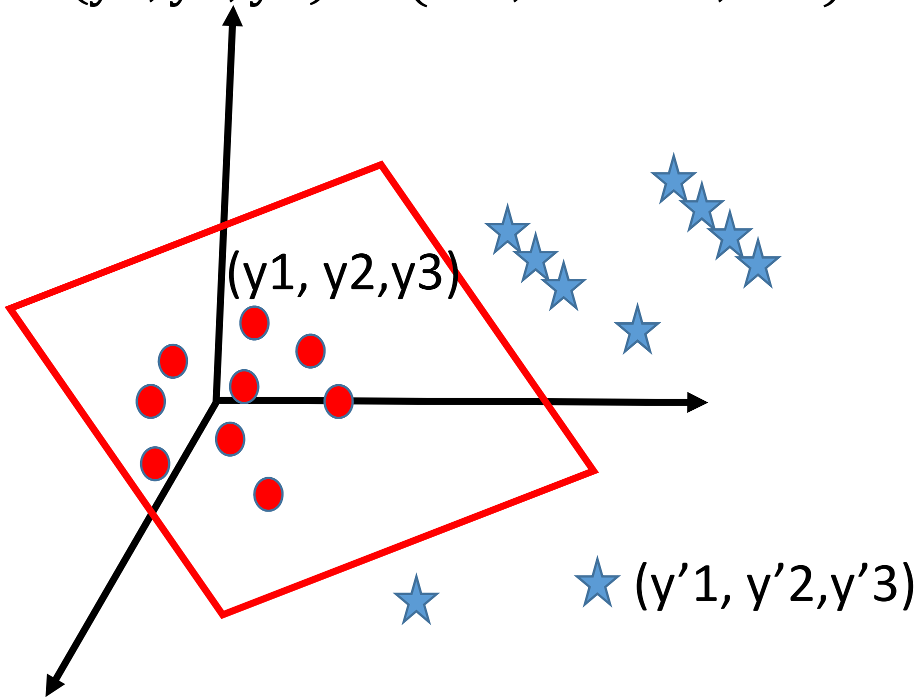
derivation

Primary Philosophy

Feature Map: φ(x)

$$(x1, x2) \rightarrow (y1, y2, y3) = (x1^2, \sqrt{2}x1x2, x2^2)$$



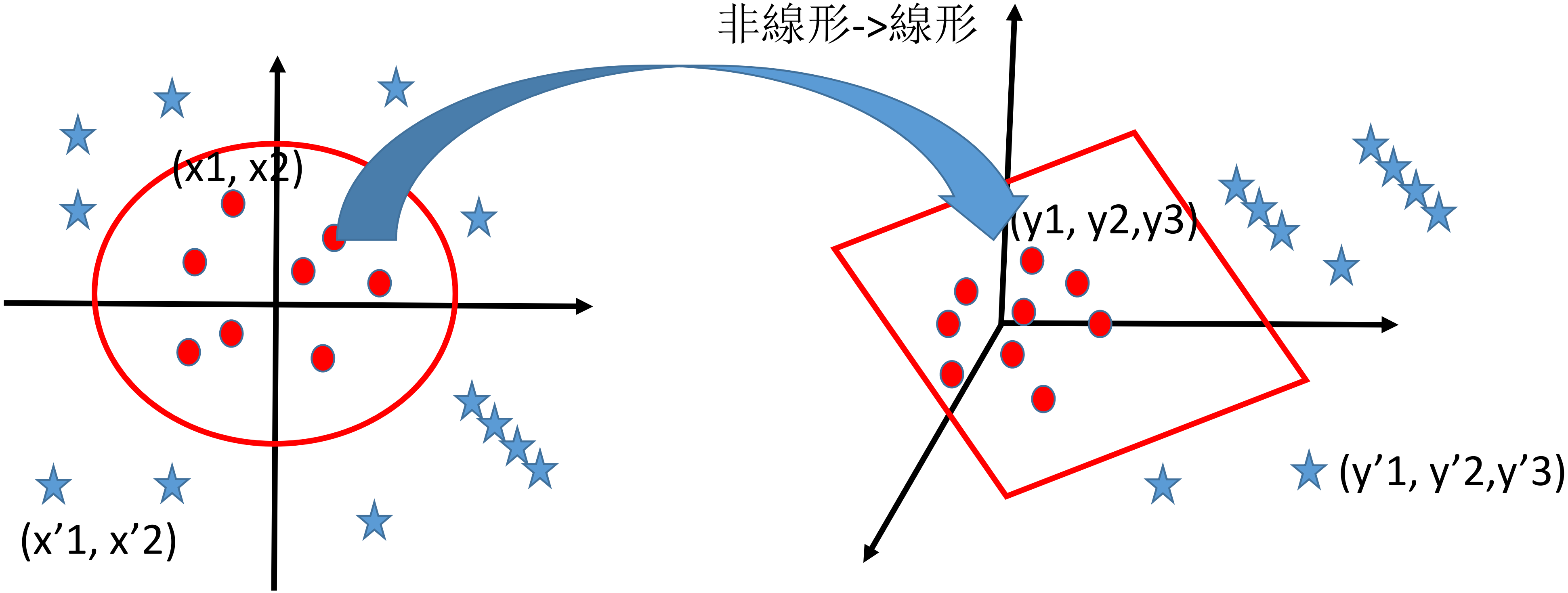


$$\frac{x1^2}{a^2} + \frac{x2^2}{b^2} = 1 \rightarrow (y1,y2,y3)=?$$

Primary Philosophy

R^2→R^3, nonlinear → Linear

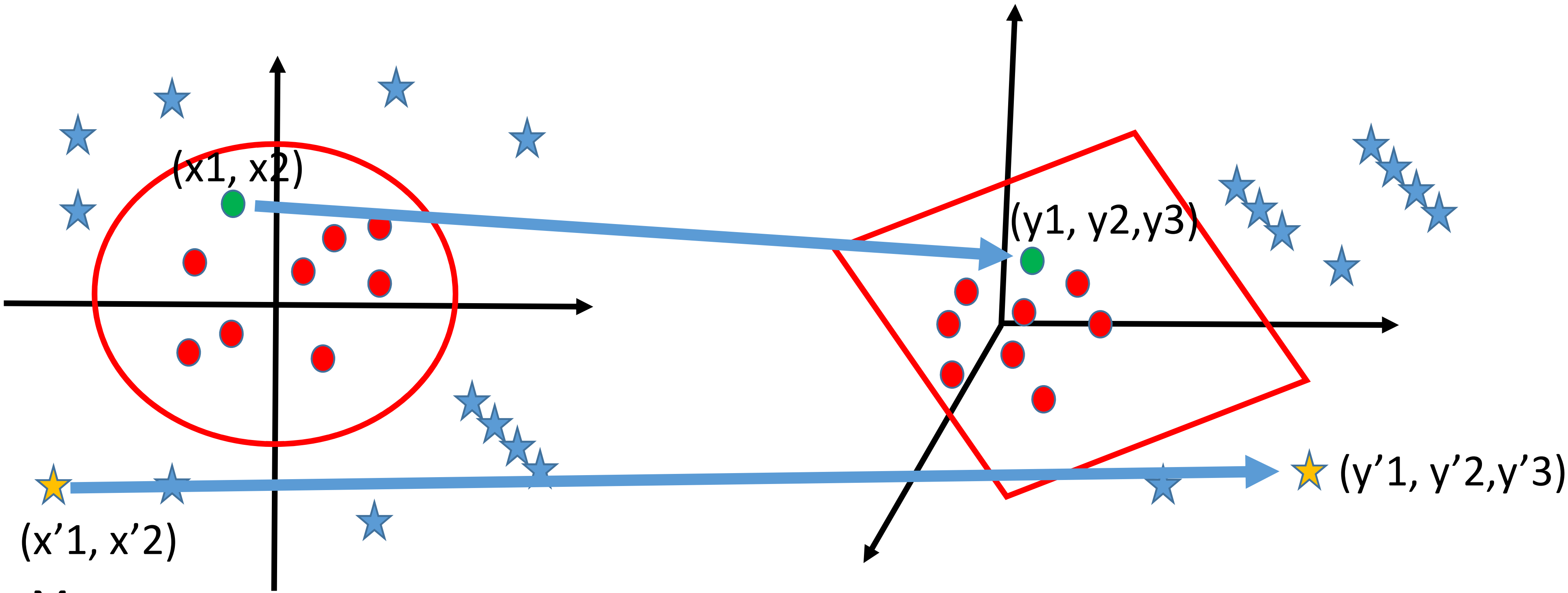
非線形->線形



$$\frac{x1^2}{a^2} + \frac{x2^2}{b^2} = 1 \rightarrow \frac{y1}{a^2} + \frac{y3}{b^2} = 1$$

derivation

Inner Product and Kernel Function



Feature Map:

$(x1, x2) \rightarrow (y1, y2, y3) = (x1^2, \sqrt{2}x1x2, x2^2)$

$$< \varphi(x), \varphi(x') > = (< x, x' >)^2 = K(x, x')$$

Inner Product and Kernel Function

- Inner product in feature space = f(inner product in original space)
- Explicit projection is not necessary
- Kernel function
  - Embedding data in a vector space (high D)
  - Looking for linear relationship

- Question: What is/are the important property(ies)/ metric(s), after we project original space into high dimensional space?

derivation

Distance and Angle

- Distance in feature space

$$|\varphi(x) - \varphi(x')|^2 = \ldots$$
$$= \kappa(x, x) - 2\kappa(x, x') + \kappa(x', x')$$

- What is the …?

derivation

Distance and Angle

- Angle in feature space

$$\cos\theta = \frac{\kappa(x, x')}{\sqrt{\kappa(x, x)}\sqrt{\kappa(x', x')}}$$

- How to compute?
- Hint $<\varphi(x), \varphi(x')> = |\varphi(x)| * |\varphi(x')| * \cos\theta$

derivation

A simple classifier in feature space

Problem:

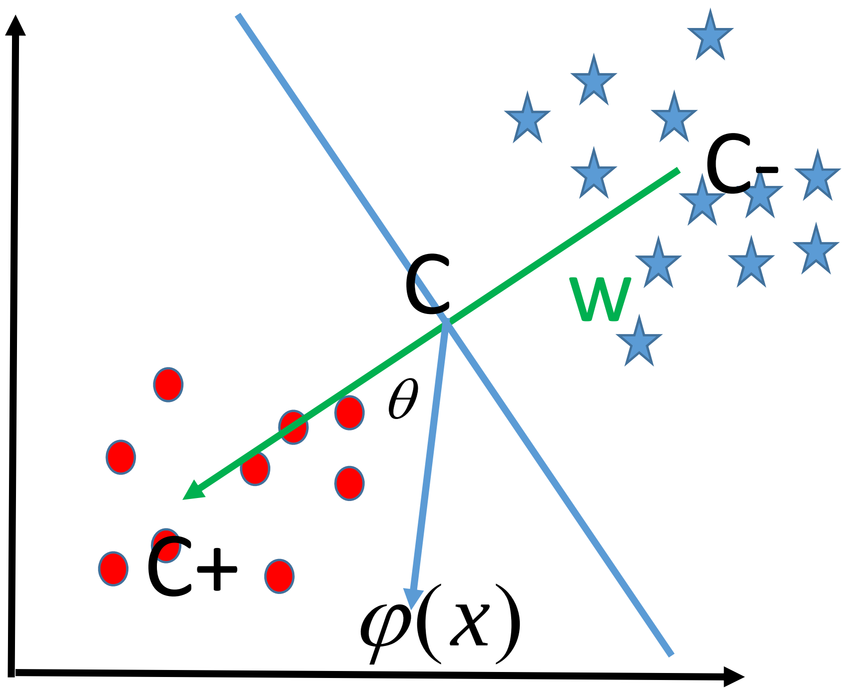
$$\{(x1, y1)(x2, y2)...(xn, yn)\} \subset R^d \times \{+1, -1\}$$

$$\{(\varphi(x1), y1)(\varphi(x2), y2)...(\varphi(xn), yn)\} \subset H \times \{+1, -1\}$$

Follow the items:

- Center point of cycle?
- Center point of star?
- Vector between cycle and star
- Cos θ =?
- How to judge to category?

- Is there any other method? (Exercise)



Kernel Matrix and Feature Map

- Feature map is necessary?
- Can we only use Kernel function?
- What Kernel can be used?
- Given a feature map, can we find a kernel to compute inner product in feature space?
- Given a kernel function, can we construct a feature space, where using kernel function to compute inner product?

Kernel Matrix and Feature Map

- Kernel Matrix

$$\kappa = \begin{pmatrix} \kappa(x_1, x_1) & \dots & \kappa(x_1, x_n) \\ \vdots & \ddots & \vdots \\ \kappa(x_n, x_1) & \cdots & \kappa(x_n, x_n) \end{pmatrix}$$

- Kernel Matrix should be finitely positive semi-definite matrix

$$\langle x, Ax \rangle = x^T A x \geq 0$$

derivation

Kernel Matrix and Feature Map

- Problem: Proof linear kernel is finitely positive semi-definite matrix

$$\kappa(x,z) = \langle x, z \rangle$$

Kernel Matrix and Feature Map

- Some Kernel function
  - Linear Kernel

$$\kappa(x,z) = \langle x, z \rangle$$

  - Polynomial Kernel

$$\kappa(x,z) = (\langle x, z \rangle + 1)^r$$

  - Gaussian Kernel

$$\kappa(x,z) = \exp\left(\frac{-|x-z|^2}{2\sigma^2}\right)$$

  - Laplacian Kernel

$$\kappa(x,z) = \exp\left(\frac{-|x-z|}{\sigma}\right)$$

Exercise

- Implementing simple classifier in this lecture by Matlab, C, C++ or Java…
- In the classifier, we use angle to judge the new data’s category:
  - Is there any other method to judge it.
  - If so, please write the formal algorithm (equation).