Generalized Discriminant Analysis

Yan PEI/裴岩 /ペイ　イエン

peiyan@u-aizu.ac.jp

Generalized Discriminate Analysis

- Notation
  - L : # of classes
  - N_i: # of samples in class i
  - N: # of all samples
  - $\phi(x_j^{(i)})$: the j-th sample in class I

$$X_i^T = [\phi(x_j^{(i)}), \phi(x_j^{(i)}), \ldots, \phi(x_j^{(i)})]$$

$$X^T = [X_1^T, X_2^T, \ldots, X_L^T]$$

- E.g.
  - i: X^(i)_1, X^(i)_2, …, X^(i)_N_i

Generalized Discriminate Analysis --objective function

- What is the two objectives of Linear discriminate analysis?
  - (1) Minimize distance of data as smaller as possible (mean in one class)
  - (2) Maximize distance of each group as bigger as possible (mean between each class)

- Suppose M_0=0

$$S_b^{GDA} = \sum_{i=1}^{L} \frac{N_i}{N} (m_i)(m_i)^T$$

$$S_w^{GDA} = \sum_{i=1}^{L} \sum_{j=1}^{N_i} \frac{1}{N} \phi(x_j^{(i)}) \phi(x_j^{(i)})^T$$

derivation

Generalized Discriminate Analysis --objective function solution

- 1 what is m_i
- 2 what is S_b
- 3 what is S_w

$$m_i = \frac{1}{N} X_i^T \bullet 1_{N_i \times 1} \qquad B_i = \frac{1}{N_i} \bullet 1_{N_i \times 1}$$

$$S_b^{GDA} = \frac{1}{N} X^T B X \qquad B = \begin{bmatrix} B_1 & & 0 \\ & \ddots & \\ 0 & & B_L \end{bmatrix}$$

$$S_w^{GDA} = \frac{1}{N} X^T X$$

derivation

Generalized Discriminate Analysis

$$S_b^{GDA} = \frac{1}{N} X^T B X$$

$$S_w^{GDA} = \frac{1}{N} X^T X$$

- Eigen value problem

$$S_b^{LDA} v = \lambda S_w^{GDA} v$$

$$B = \begin{bmatrix} B_1 & & 0 \\ & \ddots & \\ 0 & & B_L \end{bmatrix}$$

$$(\frac{1}{N} X^T B X) v = \lambda (\frac{1}{N} X^T X) v$$

$$X^T B X \bullet X^T \alpha = \lambda (X^T X) \bullet X^T \alpha$$

$$X X^T B X X^T \alpha = \lambda X X^T X X^T \alpha$$

$$B_i = \frac{1}{N_i} \bullet 1_{N_i \times 1}$$

$$\kappa B \kappa \alpha = \lambda \kappa \kappa \alpha$$ Find alpha from this equation

New data coming:

$$v^T \varphi(x) = (X^T \alpha)^T \varphi(x) = \alpha^T X \varphi(x) = \alpha^T \begin{bmatrix} \varphi(x_1) \\ \vdots \\ \varphi(x_N) \end{bmatrix} \varphi(x) = \alpha^T \begin{bmatrix} \kappa(x_1, x) \\ \vdots \\ \kappa(x_N, x) \end{bmatrix}$$