Performance analysis of two-dimensional optical code-division multiple-access systems using novel multi-code pulse-position modulation

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Abstract: Previous works show that pulse-position modulation (PPM) is an effective signalling method for mitigating multiple-access interference, and hence is able to increase the number of users in two-dimensional (2-D) optical code-division multiple-access (OCDMA) systems. However, in order to achieve high bit-rates, 2-D OCDMA systems using PPM signalling require very high transmitted power because of the negative impact of dispersion. In this study, the authors propose a novel modulation technique of multi-code PPM (MCPPM), which is the combination of the PPM and multi-code modulation (MCM). As the proposed technique inherits advantages from both MCM (in mitigating the dispersion) and PPM, 2-D OCDMA systems using MCPPM signalling are able to support higher user bit-rates for a larger number of users at low transmitted powers. Numerical results show that 2-D OCDMA systems using 4-4-MCPPM can support 60 users with 5 Gbps per user at the transmitted power of $-7 \text{ dBm}$. The power gain in this case is 11 dB compared to 2-D OCDMA systems using 4-PPM signalling. The proposed systems using 4-2-MCPPM can support as many as 36 users with the bit-rate of 10 Gbps per user and transmitted power is $-2 \text{ dBm}$.

1 Introduction

Recently, optical fibre communications has been playing an increasingly important role in the domain of access networks because of the rapid growth of the Internet and broadband services. To reduce cost and to make use of the vast bandwidth provided by optical fiber, multiplexing more than one user on a single optical fibre is desired. The present optical access networks are deployed on passive optical networks (PONs) and employ time-division multiplexing (TDM) for the physical layer. However, because of the nature of time multiplexing, the total throughput of a TDM-based system is limited by the electronic processing. Wavelength-division multiplexing (WDM) is technically an attractive technique that can accommodate large number of users with high-speed connections. However, the introduction of WDM to the access environment is still limited because it requires expensive components and strict wavelength managements. Optical code-division multiple-access (OCDMA), which offers a number of advantages including asynchronous access, scalability, inherent security and especially the potentially lower cost than WDM, has therefore emerged as an alternative solution for the next-generation broadband optical access networks [1].

Early works on OCDMA technique focused on one-dimensional (1-D) systems, which use only either time [2] or frequency domain [3, 4] for signal encoding. Recently, two-dimensional (2-D) OCDMA systems, whose the signal is encoded using both time and frequency domains, have been proposed and received even more interests [5, 6]. Compared to the 1-D codes, the major advantages of 2-D codes are the bigger cardinality and lower cross-correlation. As a result, the system performance, in terms of both bit-error rate (BER) and the maximum number of users, can be significantly improved.

Previous works have shown that multiple-access interference (MAI), optical beating interference (OBI) and group velocity dispersion (GVD) are the main factors that degrade the performance of 2-D OCDMA systems [7–10]. MAI and OBI limit the number of users whereas GVD limits the system’s transmission length. Several methods have been proposed to mitigate the negative impact of MAI and OBI. To reduce MAI, $M$-ary PPM (M-PPM) is recommended as an efficient method [11]. PPM is more useful when it is combined with optical hard-limiter or heterodyne detection, effective techniques to relax OBI [8, 12]. These techniques, however, cannot reduce the impact of GVD, which is even stronger in PPM system. This is because the chip rate for PPM systems is considerably high and therefore places a stringent limitation to the user bit rate. On the contrary, $M$-ary pulse amplitude modulation (M-PAM) can be utilised to decrease the chip rate hence reduce the impact of GVD. However, the performance of
OCDMA systems using PAM is drastically deteriorated because of MAI pulses with high intensities even when the number of MAI pulses is small [9].

Recently, M-ary multi-code keying or multi-code modulation (MCM) has been realized as a good solution for GVD problem in OCDMA systems [13, 14]. Similar to PPM, MCM is also a M-ary modulation scheme, thus is able to mitigate MAI as well. However, instead of using different pulse positions, multiple codes are used to encode M-ary symbols in the MCM signalling method. Therefore in the OCDMA systems using MCM signalling, the pulse width is larger than that of the ones using PPM; as a result, the impact of GVD can be reduced. However, it is also shown that the anti-MAI ability of MCM systems is not as good as that of PPM ones [14].

To simultaneously mitigate the impact of both GVD and MAI, we propose to combine MCM and PPM to create a novel modulation scheme, and name it multi-code PPM (MCPPM). MCPPM is therefore characterised by two parameters, the number of codes \(M_c\) and the number of positions \(M_p\). An illustration of the proposed \(M_c\)--\(M_p\)-MCPPM scheme against PPM and MCM is shown in Fig. 1 where two codes (i.e. \(M_c = 2\)) and two positions (i.e. \(M_p = 2\)) are used to create 2-2-MCPPM. In the conventional systems using PPM signalling, a high level of modulation causes the reduction of pulse width that consequently limits the bit rate because of the effects of dispersion. In the MCM systems, higher level of modulation requires more number of codes for symbol encoding, and therefore limits the number of users [14]. In the newly proposed MCPPM signalling, those problems can be effectively addressed and it is possible to increase the level of modulation to support higher user bit-rates for a large number of users at low transmitted power levels. To further enhance the performance of 2-D OCDMA systems using MCPPM signalling, we also propose to use heterodyne detection receiver so that the receiver sensitivity can be improved and the number of users can be further increased [8].

The rest of the paper is organised as follows. Section 2 presents the principle of modulation schemes and OCDMA encoding process. System descriptions and performance analysis are presented in Section 3 and 4, respectively. Section 5 shows the numerical results and discussion. Finally, Section 6 concludes the paper.

## 2 Modulation and OCDMA encoding

### 2.1 Modulation schemes

In this subsection, we present the principle and the comparison of three modulation schemes including PPM, MCM and MCPPM. The bit duration \((T_b)\) are used as benchmark for the comparison.

In PPM, blocks of \(\log_2 M_p\) binary data are first mapped into \(M_p\)-ary symbols denoted as \(s_u (u = 0, \ldots, M_p - 1)\). The optical pulse is then position modulated to one of \(M_p\) disjoint time slots corresponding with the \(M_p\)-ary symbols. Each PPM frame length of \(T_b \log_2 M_p\) contains \(M_p\) time slots, the duration of each time slot (or symbol duration) is hence written as \(T_{\text{PPM}} = (T_b \log_2 M_p)/M_p\).

For MCM, blocks of \(\log_2 M_c\) binary data are also mapped into \(M_c\)-ary symbols. However, unlike PPM, each symbol in MCM is directly encoded by an OCDMA code instead of a position. As a result, the duration of MCM symbol \(T_{\text{MCM}} = T_b \log_2 M_c\), which is \(\log_2 M_c\) times wider than the bit duration. The negative impact of GVD hence will be reduced.

In MCPPM, each symbol is simultaneously modulated by a couple of position and an OCDMA code. The symbols that have the same position will have different codes and vice versa. The number of symbols \(M\) is related to \(M_c\) and \(M_p\) by \(M = M_c \times M_p\). It is worth noting that PPM and MCM are special cases of MCPPM when \(M_c = 1\) and \(M_p = 1\), respectively. The duration of \(M_c\)--\(M_p\)-MCPPM symbol hence can be expressed as \(T_{\text{MCPPM}} = (T_b \log_2 M)/M_p\).

### 2.2 OCDMA encoding

In 2-D OCDMA systems that use different modulation schemes such as PPM, MCM or MCPPM, each symbol is spread into a chip sequence by a specific 2-D code, which is the combination of a time-spreading (TS) pattern and a wavelength-hopping (WH) pattern. The chip sequence includes chip ‘1’s’ and ‘0’s’. A chip ‘1’ is corresponding to an optical pulse while a chip ‘0’ means no pulse. The positions of chip ‘1’s’ and ‘0’s’ are determined by the TS pattern while the wavelengths of chip ‘1’s’ are determined by the WH pattern. The length of the chip sequence is equal to the code length \((N)\). The chip duration is therefore \(N\) times narrower than the symbol duration.

In this paper, we use prime code for both TS and WH patterns. A TS pattern can be generated using the linear congruent placement operator, to place a pulse within a block as follows

\[
a_{xy} = [x, y], \quad x, y = 0, 1, \ldots, p_s - 1
\]

where \(p_s\) is a prime number, and \([\cdot]\) denotes modulo \(p_s\) operation [1]. The value \(a_{xy}\) determines the position of the only pulse in the block \(x\)th (in \(p_s\) blocks of a \(p_s\)-length code) of pattern \(y\)th (in total of \(p_s\) spreading patterns). Similarly, each WH pattern can also be generated from a prime number \(p_h\) \((p_h \leq p_s)\), which is the number of wavelengths used for the 2-D code. In case of the WH, the result of the \(p_h\) modulo operation determines the wavelength number (in total \(p_s\) wavelengths).

An example of the process of generating the TS and the WH patterns is illustrated in Table 1. A 2-D prime code that is constructed from \(S_0H_1\) is \(\lambda_00000000\lambda_00000000\lambda_00000000\lambda_00000000\).

Note that the WH pattern \(H_0\) has only one wavelength, as seen in Table 1, this hopping pattern is thus not used. The number of WH patterns is therefore \(p_h - 1\), while the number of TS patterns is \(p_s\). As a result, a 2-D code set that consists of \(p_h(p_h - 1)\) distinctive 2-D prime codes of length \(p_s\) can be generated. The code weight is \(p_s\) and the
maximum cross-correlation between two 2-D codes is one [5].

3 System descriptions

A schematic diagram of a 2-D OCDMA system using MCPPM signalling and heterodyne detection is shown in Fig. 2. The system supports \( K \) users, which are corresponding to \( K \) pairs of transmitter and receiver. For the illustrative purpose, one transmitter and one receiver are depicted in detail in the figure. Signal from all transmitters are combined by a combiner and sent to receiver side through an optical fibre, which is modelled as a linear dispersive optical channel. At the receiver side, a splitter is used to deliver received signal to all receivers. Following subsections will describe in detail the principle of each part of the system.

3.1 MCPPM transmitter

At the transmitter, a symbol converter is used to map each block of \( m \)-bit into one symbol, denoted as \( s_u \) (\( u = 0, \ldots, M-1 \)). Each symbol is then encoded by a pair of time slot and 2-D prime code, which is specified to this symbol. The encoder, as shown in Fig. 3a, can be implemented using fibre Bragg grating (FBG) arrays, whose wavelengths are tuned for WH pattern of the 2-D code. \( M \) FBG arrays are connected to an encoding controller, which consists of a 1: \( M \) splitter/combiner and \( M \) on/off optical switches controlled by \( M \) electrical inputs. An input symbol \( (s_u) \) controls its optical switch (SW) to select the corresponding FBG array. The encoded signal departing from a transmitter as illustrated in Fig. 3b is combined with signals at a combiner. To compensate splitting loss and insertion loss caused by the encoders/decoders as well as the combiner/ splitter that is used for optical distribution network, an optical amplifier is placed just after the combiner [15, 16]. Finally, amplified signal is sent to the receiver side through an optical fibre.

3.2 Linear dispersive optical channel

During propagation over the fibre, optical pulses are affected by fibre attenuation and chromatic dispersion. To analyse their effects, each optical pulse is modelled as a Gaussian pulse and optical fibre is considered as a linear dispersive channel. After propagating over the optical fibre of length \( L \) kilometre, the amplitude of received Gaussian pulse can be expressed as [17]

\[
A_L(t) = \sqrt{P_0} e^{-aL} \frac{T_0}{(T_0^2 - j\beta_2 L)^{1/2}} e^{-\frac{t^2}{2(T_0^2 - \beta_2 L)}}
\]

where \( \alpha \) and \( \beta_2 \) are the attenuation and dispersion coefficients of the fibre, respectively. \( T_0 \) is half-width of the pulse (at 1/e intensity point). \( P_0 \) is the transmitted peak power of optical pulse, which is assumed to be the same for all users. For a fair comparison with other systems, the analysis is
considered under a constraint on the average power per bit denoted as \( P_b \). The relation between \( P_0 \) and \( P_b \) is given by [14]

\[
P_0 = \frac{T_s \log_2 M}{T_0 P_b \sqrt{\pi}} P_b
\]  

(3)

The average received power per chip, which has been considered the insertion losses and amplifier gain \((G)\), can be written as

\[
P_s = \frac{G}{M^2 K^2 L_c^2} \int_{-T_c/2}^{T_c/2} |A_2(t)|^2 dt
\]  

(4)

where \( L_c \) is the insertion loss of each coupler (splitter/ combiner). As the received signal passes through five coupler, the total loss is \( L_c^5 \). Splitting loss because of the optical splitters at the encoder and decoder is \( M^5 \) while it is \( K \) for the optical distribution network.

### 3.3 MCPPM receiver using heterodyne detection

At the receiver, the received signal including MAI is first mixed coherently with a local oscillator (LO) whose power per wavelength is \( P_l(LO) \). The mixed signal is then decoded at the FBG-based MCPPM decoder (Fig. 3c), which consists of \( M \) decoded branches corresponding to \( M \) symbols. At each branch, the decoded signal is converted into an electrical signal (at intermediate frequency, IF) by a photodetector (PD). At the PD, OBI will occur because of the beating between pulses with the same wavelength. A heterodyne demodulator (HD), which consists of a band-pass filter, a demodulator, and a low-pass filter, is then used to change the IF signal into a baseband one through a synchronous demodulation [17]. The symbol detector has \( M \) inputs corresponding to \( M \) decoded branches. The currents at its \( M \) inputs are compared in parallel to determine the input that has the largest value. The detected symbol will be decided correspondingly to this input. Finally, the detected symbol is converted into the binary data.

### 4 Performance analysis

In the following analysis, we assume bit and chip streams from all users are to be synchronised, that is, the upper bound performance, for all users. Also, the polarisation between the optical signals and the LO are assumed to be matched, the effect of polarisation mismatch is thus ignored. Moreover, to focus on the impact of MAI, OBI and GVD, which are dominant compared to other noise and interference [7, 10], on the performance of the new MCPPM method against conventional ones, we assume the ideal optical amplifier and thus ignore amplification noises in the analysis.

In the proposed system, errors happen when the transmitted symbols are wrongly detected. Denote \( P_e \) as the symbol error rate, the BER can be derived as [8]

\[
\text{BER} = 0.5 MP_e / (M - 1)
\]  

(5)

where \( M \) is level of modulation.

Without loss of generality, we assume that symbol \( s_0 \) is transmitted. Let \( I_u \) denote the photocurrent at the \( u \)th input of the symbol detector \((u \in \{0, \ldots, M - 1\})\), employing a union bound, \( P_e \) can be given as

\[
P_e \leq 1 - \text{Pr}\{I_u > I_{u^*} | u \in \{1, \ldots, M - 1\}, s = s_0\}
\]

\[
\leq \sum_{u=1}^{M-1} \text{Pr}\{I_u \geq I_0 | s = s_0\}
\]

\[
= (M_c - 1) \text{Pr}\{I_u^{(s)} \geq I_0 | s = s_0\}
\]

\[
+ M_p(M_c - 1) \text{Pr}\{I_u^{(p)} \geq I_0 | s = s_0\}
\]  

(6)

where \( s \) represents the transmitted symbol. The \( I_u^{(s)} \) is the photocurrent of the symbol that has the same position with that of \( s_0 \), whereas the \( I_u^{(p)} \) represents the photocurrent of the symbol that has different position with that of \( s_0 \). For example, in the case of 2-2-MCPPM (i.e. \( M_c = M_p = 2 \), \( I_u^{(s)} \) is \( I_2 \) while \( I_u^{(p)} \) is \( I_1 \) or \( I_3 \).

Denote \( \kappa_u \) as a random variable that represents the number of users whose transmitted symbol \( s = u \), it can be modelled as a binomial variable. Equation (6) can be written as

\[
P_e \leq (M_c - 1) \sum_{l_p=0}^{K-1} \sum_{l_{\kappa_p}=0}^{K-1-l_p} \text{Pr}\{\kappa_p = l_{\kappa_p}, \kappa_p = l_p\}
\]

\[
\times \text{Pr}\{I_u^{(p)} \geq I_0 | s = s_0, l_p, l_{\kappa_p}\}
\]  

(7)

where, \( K \) is the number of simultaneous users. \( l_p \) is the number of interfering users sending the symbol that has the same position with that of \( s_0 \) and \( l_{\kappa_p} \) is the number of interfering users sending the symbol that different position with that of \( s_0 \). The probabilities \( \text{Pr}\{\kappa_p = l_{\kappa_p}\} \) and \( \text{Pr}\{\kappa_p = l_p, \kappa_p = l_p\} \) can be expressed as

\[
\text{Pr}\{\kappa_p = l_p\} = \binom{K - 1}{l_p} \left( \frac{1}{M_p} \right)^{l_p} \left( 1 - \frac{1}{M_p} \right)^{K - 1 - l_p}
\]  

(8)

and

\[
\text{Pr}\{\kappa_p = l_p, \kappa_p = l_p\} = \binom{K - 1}{l_p} \left( \frac{1}{M_p} \right)^{l_p} \left( 1 - \frac{1}{M_p} \right)^{K - 1 - l_p}
\]

\[
\times \left( \frac{K - 1 - l_p}{M_p - 1} \right)^{l_p} \left( M_p - 1 \right)^{K - 1 - l_p - l_p}
\]

\[
\times \left( 1 - \frac{1}{M_p - 1} \right)^{K - 1 - l_p - l_p}
\]  

(9)

At each decoded branch, a pulse from interfering users becomes a MAI one and contributes to the photocurrent at the input of the symbol detector when it matches both position and wavelength with the corresponding decoder. Denote \( \zeta_u \) as the random variable representing the total number of MAI pulses of the \( u \)th input, \( \zeta_u \) can be modelled as a binomial random variable with probability \( (\mu_\lambda / p^2_u) \) where \( \mu_\lambda \) is the average number of wavelengths common to a pair of 2-D codes [7]. Therefore the probability that there are \( m_u \) MAI pulses when the total number of interfering
users is \( I_a \) can be given as

\[
Pr\{\xi_u = m_u|I_a\} = \left( \frac{I_u}{m_u} \right)^{\frac{n}{2}} \left( 1 - \frac{I_u}{m_u} \right)^{\frac{n}{2} - m_u}.
\]

(10)

Based on (10), \( Pr\{I_a^{(p)} \geq I_0|s = s_0, I_p\} \) and \( Pr\{I_a^{(p)} \geq I_0|s = s_0, I_p, f_p\} \) can be calculated as

\[
Pr\{I_a^{(p)} \geq I_0|s = s_0, I_p\} = \sum_{i=0}^{l_p} Pr\{\xi_p = i|l_p\} \sum_{j=0}^{l+1} Pr\{\xi_p = j|l_p\} Q\left( \frac{I_0 - I_a^{(p)}}{\sqrt{\sigma_0^2 + \sigma_p^2}} \right)
\]

(11)

\[
Pr\{I_a^{(p)} \geq I_0|s = s_0, I_p, f_p\} = \sum_{i=0}^{l_p} Pr\{\xi_p = i|l_p\} \sum_{j=0}^{l_p} Pr\{\xi_p = j|l_p\} Q\left( \frac{I_0 - I_a^{(p)}}{\sqrt{\sigma_0^2 + \sigma_p^2}} \right)
\]

(12)

Here, \( Q(x) \) is the \( Q \) function. Owing to using heterodyne detection, \( I_0 - I_a^{(p)} = R_s \sqrt{2P_T L} (p_s + i - j) \), where \( P_s \) is the autocorrelation peak (or code weight) of 2-D prime code. The total variance of noise and interference contributed from the 0th and the \( n \)th input can be expressed as

\[
\sigma_0^2 + \sigma_p^2 = \frac{2R_s B_c}{B_0} R_s^2 P_s^2 \left[ i + \frac{1}{p_s} \left( \frac{i}{2} \right) + \frac{1}{p_s} \left( \frac{j}{2} \right) \right] + 2\sigma_{r}.\]

(13)

where the first term of (13) represents the OBI while the second term, \( \sigma_{r}^2 \), is the receiver noise, which is the total of the shot noise (\( \sigma_{s}^2 = 2eR_s P_s L \sqrt{B_c} \)) and thermal noise (\( \sigma_{th}^2 = 8nkT_p B_c \)) [8]. Here, \( e \) is the electron charge, \( B_c \) and \( B_0 \) are the electrical and optical bandwidths, respectively, \( k_0 \) is Boltzmann’s constant, \( T_0 \) is the receiver noise temperature and \( C \) is the receiver capacitance.

It is worth noting that PPM is a special case of MCPPM with \( M = 1 \), that is, without multi-code modulation. In this case, the first term of (6) is omitted in the calculation of \( P_e \). Similarly, MCM is a special case of MCPPM with \( M_p = 1 \), that is, without position modulation. The second term of (6) is hence excluded in the calculation of \( P_e \).

5 Numerical results

In this section, we first show the limitations of conventional modulation schemes, including on-off keying (OOK), PPM, and MCM. Next, the system improvement of 2-D OCDMA systems using MCPPM is revealed. Finally, we investigate the network capacity with different levels of modulation, including the combinations of different number of codes and positions.

We use the 2-D prime code set with \( p_s = 11 \) and \( p_h = 37 \). As the proposed system is considered for access networks, the transmission length \( L = 20 \) km is chosen. Other system parameters are shown in Table 2. To have a fair comparison with other systems, we assume that all losses, including insertion and splitting loss, are compensated by optical amplifier, that is, \( G = M^2 KL_c^2 \).

We first investigate BER against the average transmitted power per bit (\( P_b \)) when \( R_b = 2.5 \) Gbps. The numerical result is obtained from (5), which presents the relation between BER and the symbol error rate, \( P_e \). By changing the average transmitted power per bit and the number of simultaneous users, BER can be investigated accordingly to the change of \( P_e \).

Fig. 4 confirms that, the performance of 2-D OCDMA systems using PPM and MCM is improved in comparison with the system using OOK. In addition, when the number of users is small (e.g. \( K = 25 \)), the performance of MCM system is better than PPM one since it requires lower value of transmitted power. This is because MCM system is able to suppress the negative impact of GVD. However, when \( K \) is large enough, for example, \( K = 50 \) users, the impact of MAI and OBI becomes severe, PPM system is still able to keep the BER floor below \( 10^{-9} \) with high transmitted power while MCM one is not. This shows that MAI and OBI are better relaxed in the PPM system.

Next, Fig. 5 shows the average transmitted power per bit (\( P_b \)) against the number of supportable users (\( K_{max} \)) when \( R_b = 5 \) Gbps. \( K_{max} \) is defined as the number of users that the systems can support with the BER of \( 10^{-9} \). It is seen that, at the bit rate of 5 Gbps or above, the conventional 2-D OCDMA systems using either PPM or MCM cannot simultaneously achieve large \( K_{max} \) and low \( P_b \). More specially, \( M \)-MCM systems cannot support the number of users that is equal to or larger than 30 users. This is

**Table 2 System parameters**

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD responsivity</td>
<td>( R )</td>
<td>1</td>
</tr>
<tr>
<td>LO optical power</td>
<td>( P_{LO} )</td>
<td>0 dBm</td>
</tr>
<tr>
<td>attenuation coefficient</td>
<td>( \Lambda )</td>
<td>0.2 dB/km</td>
</tr>
<tr>
<td>dispersion coefficient</td>
<td>( \beta_s )</td>
<td>20 ps²/km</td>
</tr>
<tr>
<td>bit rate per user</td>
<td>( R_b )</td>
<td>Gbps</td>
</tr>
<tr>
<td>electrical bandwidth</td>
<td>( B_e )</td>
<td>( R_b \log_2 M ) Hz</td>
</tr>
<tr>
<td>chip duration</td>
<td>( T_c )</td>
<td>log₂(( M/(R_b P_s^2) )) s</td>
</tr>
<tr>
<td>half-width of pulse</td>
<td>( T_u )</td>
<td>( T_c/(B \sqrt{\ln 2}) ) s</td>
</tr>
</tbody>
</table>

**Fig. 4 BER against the average transmitted power per bit (\( P_b \)) when \( R_b = 2.5 \) Gbps, and \( L = 20 \) km**
because the increase of modulation level (i.e. $M$) only helps to reduce $P_b$. On the other hand, PPM systems is not able to reduce $P_b$ to $0 \, \text{dBm}$ regardless of using $4$-PPM. It is worth noting that, in PPM systems, there is no means of using $M > 4$, because the system performance is decreased because of the reduction of pulse width [14].

It is clearly shown in Fig. 6 that the performance of 2-D OCDMA systems is significantly improved by using MCPPM. The number of supportable users that is larger than 30 users can be supported with the required transmitted power is below $0 \, \text{dBm}$. For example, when $K_{\text{max}} = 60$ users, the required transmitted power per bit in 4-4-MCPPM systems is only $-7 \, \text{dBm}$, which gains $11 \, \text{dB}$ compared to that of 4-PPM system. The reduction of transmitted power helps to increase the bit rate and avoid using additional amplifier, which results in high cost. Large $K_{\text{max}}$ and low $P_b$ are favourable features for optical broadband access networks.

In order to obtain useful information for system design, we investigate the maximum user bit rate ($R_{\text{bmax}}$) that is computed at BER $= 10^{-9}$ against the number of simultaneous users. The transmitted power per bit is fixed at $-2 \, \text{dBm}$. In Fig. 7, the numerical results are evaluated for several types of MCPPM, including $2$-$2$-MCPPM, $2$-$4$-MCPPM, $4$-$2$-MCPPM and $4$-$4$-MCPPM. It is seen that, when $K \leq 43$ users, $4$-$2$-MCPPM should be chosen as it can guarantee the bit rate of at least $8.5 \, \text{Gbps}$ per user. When $K > 43$ users, $4$-$4$-MCPPM should be used, because the effect of MAI and OBI becomes severe.

Finally, based on previous result, we analyse the network capacity ($C$) against the number of users in Fig. 8. The network capacity is defined as the product of the number of users and the maximum user bit rate. In the cases of using two positions for modulation (i.e. $2$-$2$-MCPPM and $4$-$2$-MCPPM), there is an optimum number of users at $K = 40$ users, where the network capacity is at its maximum. The network capacity starts decreasing as $K > 40$ users because of the effect of MAI and OBI, which causes the significant reduction of user bit rate. In the cases of using $2$-$4$-MCPPM and $4$-$4$-MCPPM, the network capacity increases as the number of users increases in these cases, the reduction of user bit rate when $K$ increases is not much because four positions (i.e. 4-PPM) are used for modulation.

![Fig. 5](image1.png) Average transmitted power per bit ($P_b$) against the number of supportable users ($K_{\text{max}}$) when $R_b = 5 \, \text{Gbps}$ and $L = 20 \, \text{km}$

![Fig. 6](image2.png) Average transmitted power per bit ($P_b$) against the number of supportable users ($K_{\text{max}}$) when $R_b = 5 \, \text{Gbps}$ and $L = 20 \, \text{km}$

![Fig. 7](image3.png) Maximum user bit rate against the number of simultaneous users when $P_b = -2 \, \text{dBm}$ and $L = 20 \, \text{km}$

![Fig. 8](image4.png) Network capacity against the number of simultaneous users when $P_b = -2 \, \text{dBm}$ and $L = 20 \, \text{km}$
6 Conclusion

We have proposed the novel MCPPM technique for high bit-rate 2-D OCDMA systems. The performance of the 2-D OCDMA systems using MCPPM was theoretically analysed and compared with the ones using conventional signalling schemes. The numerical results showed that, compared to the conventional systems using PPM and MCM signalings, 2-D OCDMA systems using MCPPM were able to achieve higher user bit-rate with large number of supportable users and low transmitted power level. Especially, it was seen that, with the novel signalling scheme, 2-D OCDMA systems were able to support $36 \times 10$ Gbps users at the practical transmitted power of $-2$ dBm.

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8 References

Author Queries
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